Trigonometry:

OR THE DOCTRINE OF TRIANGLES.

First written in Latine, by

BARTHOLEMEVS PITISCVS of *Grunberg* in *Silesia*, and now Translated into English,

By *Ra: Handson.*

Wherunto is added (for the Mariners vse) certaine Nauticall Questions, together with the finding of the variation of the Compasse. All performed Arithmetically, without Mappe, Spheare, Globe, or Astrolabe, by the said R. H.

Printed for Io. Tappe.


VIRTUTE PARIT NOBILITAS
COLL: SS. TRIN:
CANT:

TO THE HONORABLE SIR THOMAS SMITH KNIGHT: AND THE RIGHT WORTHY Mr. *John Woſtenholme* Eſquire, the ſole Founders and Erectors of *Navigation* in the honorable Citie of London.

And two of the principall aduenturers of the Northweſt diſcouerie.



LD Arts are in themſelues ſo infinite, that the life of man is firſt conſumed before he come to knowe: yet the pleaſure is ſuch (eſpecially in the Mathematiques) that the more a man vnderſtandeth, the leſſe he thinkes to knowe; as ſtill couetous of more, and neuer ſatisfied. And amongſt all the Sciences Mathematicall, this Trigonometry, or dimenſion of Triangles, is copious in the contemplation of it, & more profitable in the practiſe: For thereby al heights, depths,

distances, questions of the Mappe, Globe, Sphære, or Astrolabe, may be more truly supputated, then by any Instrument whatsoeuer; besides the infinite vse thereof in Geometry, Astronomy, Cosmographie &c. Wherefore I haue aduentured thereon, as a subiect, which generally in it's owne nature, carrieth much reputation amongst the sincere louers of those Sciences: And (omitting other commendable writers in that kinde) I haue made choyse of this Diuine Mathematician, as one worthy to be well entertained, if I by attyring him in our country habit, haue not darkned that lustre wherein he first shewed himselfe: For of the many bookes that I haue seene treating of this subiect, this hath so briefly & succinctly handled the same, that I haue with much pleasure, and no lesse paines at my vacant houres, Translated him into our vulgar Tongue; hauing first entred thereupon for some priuate friends, at whose request, (and Mr. Richard Hakluyt his earnest perswasion, who hath deserued well of our Nation: It was afterwards induced to perfect and publish the same.

There are some pallats perhaps, that can relish nothing written in their owne language, any other better pleaseth their taste: such may vouchsafe the reading of the originall; neither haue I taken this paines for them, but for others of more ordi-

nary

nary dyet, who can be content with good meate though homly dreſt. To ſheild my ſelfe from thoſe ſelfe-conceited (and from ſuch as will ſooner carpe at an others worke then produce any of their owne;) I haue preſumed to make choiſe of you for my worthy Patrons, chiefly for that you before others haue beene the firſt erectors of a publike Lecture of Nauigation, for the aduancing of that honorable and neuer-too-much-admired profeſſion: whereby ordinary Sailers may become good Sea-men; and good ſea-men skilfull Mariners: which how profitable it may proue to the Common-wealth, none I am ſure but will acknowledge, and ſucceeding ages ſhall haue cauſe to record it. And albeit experience in all Artes, hath beene, and yet is holden to be the mother of learning (and that learning well deliuered is more fertill and fruitfull of children in a fewe monthes, then her mother Experience was or can be in many yeares: all which may as truly be affirmed of Nauigation, as of any other) yet in long voiages (ſuch as the Engliſh haue in our time attempted, and performed with good ſucceſſe, euen to the encompaſſing of the whole Globe of the earth) the good old woman Experience will be content, without diſdaine, to be led by her daughter Arte, the ſtaffe of her age; nor can they be diſioyned. For as he

cannot be saied to be a compleat Mariner who hath only Art; no more can he be reputed a perfect Sea-man who hath only Experience: but that man in whom both are linked together, may deseruedly be termed the Absolute Marriner fit to take charge in al voyages. With the experimental part I list not meddle, since it is common & familiar to all Saylers in their ordinary breeding: as to know the tacklings, tides, sands, soundings, harbors, lying of the land &c. but touching the Theoricke, by obseruation of the heauens, and guid-starres there, to finde a passage in the vast Ocean where no paths are, none is more necessarie for their vse, then the perfect knowledge of this Trigonometrie. And that the Excellencie thereof may be the more manifest, I haue added by way of application, an Appendix of mine owne, only for the Marriners vse, to whome I haue alwaies bene well affected. Such as it is, (though as a mouldhill to a Mountaine in comparison of the other) I willingly and humbly present vnto your Worships view, praying your gentle patronage and fauorable acceptance as from one who not deseruing, yet still desireth to be

Your euer vowed Seruant,

Raphe Handson.

To the Reader.

IN the ſecond booke teaching the making of the *Tables* of *Sines*, *Tangents* & Secants, the numbers there ſet down are not the ſame that is in the Tables printed to this booke, becauſe theſe are of a certaine Radius, to wit, 100000. and thoſe Tables expounded in the ſecond booke are of ſeuerall Radiuſſes, with the difference and increaſe of euery 10″. which was thought to be more troubleſome to the Marriners, and therefore the Tables laſt printed by Pitiſcus were omitted as ouer-tedious, his firſt being annexed herewith for the more eaſe in working. But if any man deſire thoſe Tables themſelues, they may buy them apart in the Latin printed, at Franckford 1612. And thus much I thought fit to aduertiſe, that I might not be any wayes miſtaken. In the reſt, if any defect be, thinke me as thy ſelfe. *Et humanum eſt errare.*

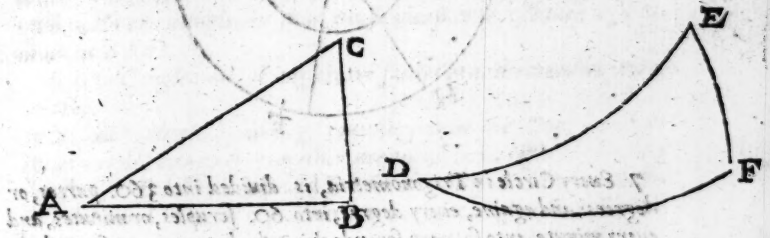
Thine RAPHE HANDSON.

THE FIRST BOOKE OF TRIGONOMETRIA: WRITTEN BY *BARTHOLMEW PITISCVS*, OF GRVNBERGE; FIRST written in Latine, and now translated into English by *Ra: Handson*, Student in the Mathematickes.

Of the Nature and quallitie of Triangles.

1 TRIGONOMETRIA is the Doctrine of the measuring of Triangles.

2 A Triangle is a figure comprehended of three sides, and three angles, as are the figures ABC and DEF.

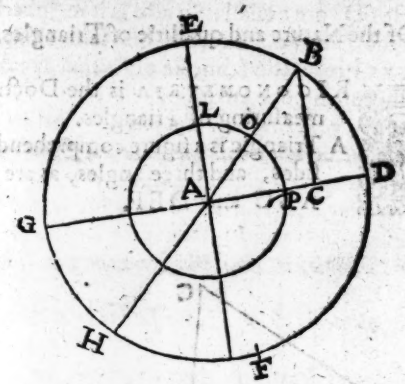


3 Euery of the 2. sides, are called the sides of the Angle, comprehended by them; the third is called the Base. As the sides A B. and A C. are your sides of the angle B A C. And B C. is the base of the said Angle.

4 *Euery side is said to subtende the angle, oppoſite vnto it.* As the ſide A B ſubtendeth your angle, A C B: The ſide A C ſubtendeth the angle, A B C, and the ſide B C, ſubtendeth the angle, B A C.

5 *The greater ſides, ſubtende the greater angles: the leſſer ſides, the leſſer angles, and equall ſides, equall angles.* The truth of the Theorem is manifeſt of it ſelfe, yet it is demonſtrated, in the 18. and 19. Pro: of the 1. booke of *Euclide*, and in the 42. and 43. Pro: of the 3. booke of *Regiomontanus*: It is alſo plainlie confirmed by the 2. Axiome of the 3. and the 3. of the 4. booke following.

6 *The meaſure of an angle, is the arche of a Circle, deſcribed from the point of the Angle, and intercepted betweene the 2. ſides, (of that Angle) increaſed.* As in the triangle A B C the meaſure of the Angle B A C is the Arche O P or B D.



7 *Euery Circle in Trigonometria, is diuided into 360. partes, or degrees: and againe, euery degree, into 60. ſcruples, or minutes, and euery minute, into ſo many ſeconds &c. Which partes are ſomuch the greater, as the Circle is greater; And thoſe arches which containe the ſame number of partes, in equall Circles, are equall; in vnequall Circles, they are ſaid to be like arches*: As the Arches B D and G H are equall. But the arches B D and O P are like arches: For example; As B D is 40. parts in the great Circle E B D; ſo is O P

40. partes

40. partes in the leſſe Circle, L O P. &c.

8 *Then A Quadrant of the ſaid Circle, is the arche of 90. partes.*

9 *The Complement of an arch, leſſe then a Quadrant is, ſo much as that arch wanteth of 90 partes.* As the Complement of the arch, B D 40. partes is the arch, B E 50. partes. And in like manner.

10 *The exceſſe of an arch greater then a Quadrant, is ſo much as the ſaid arch is more then 90. partes.* As the exceſſe of the Arch G E B 140. parts is the Arch E B 50. parts more then a Quadrant.

11 *A Semi-Circle is an Arch, of 180 partes.*

12 *The complement to a Semicircle, of an arch, leſſe then a Semicircle, is ſo much as that arch wanteth of 180. partes.* As the complement of the arch G E B 140. parts, is the arch B D 40. partes.

13 *The oppoſite angles made by croſſing of 2. lines, are equall.* As the angles B A D and G A H are equall: and likewiſe the angles G A B and H A D are equall: So alſo is it in Spherical angles. The truth of the *Theorem* appeareth of it ſelfe: Yet it is demonſtrated in the 15 Prop: of the 1. booke of *Euclide*, ſpeaking of right lines, mutually cutting one an other.

14 *An Angle is either right or oblique.*

15 *A right angle, is that whoſe meaſure is a Quadrant.*

16 *An oblique angle is either obtuſe or acute.*

17 *An obtuſe angle, is that whoſe meaſure is more then a Quadrant,* as B A G.

18 *An acute angle, is that whoſe meaſure is leſſe then a Quadrant* as B A D.

19 *The Complements of angles, are ſaid to be as the complements of Arches.*

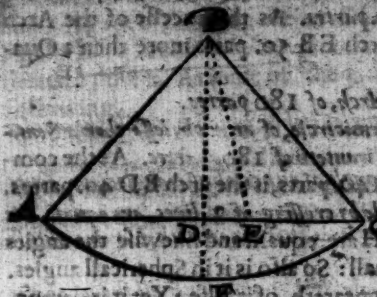
20 *All Angles comming together, vpon one line, (drawne out at length) being taken together, are equall to 2. right angles.* As the Angles B A D; E A B and E A G meeting in the point A. vpon the line G D. are equall to the 2. right angles, G A E and E A D by the operation.

21 *Therefore if 2. oblique angles, meete vpon the ſaid right line drawne out at length; th'one is the Complement of the other, to 2. right angles.* As the angle B A D is the complement of the angle, G A B to 2. right angles.

22 *A Triangle, firſt of all, hath ſome of the ſides equall, or*

elſe all the ſides vnequall.

23 *If a triangle haue ſome of the ſides equall, a perpendiculer let fall from the meeting of the equall ſides, cutteth the baſe and the angle, oppoſite to the baſe, into 2. equall partes and contrarilie.* As in the Triangle of 2. equall ſides, A B and B C the perpendiculer B D cutteth the baſe A C. and the angle, A B C. oppoſite to the Baſe, into 2. equall parts. Becauſe if it ſhould not cutte it into 2. equall partes, but ſhould fall without the middle point, D that is in E it ſhould not be perpendiculer, and ſo not the ſhortest line, betwixt the point, B. and the right line, A C. Alſo it cutteth the angle, A B C. oppoſite to the baſe, and his meaſure A E C. into 2. equall partes. Becauſe the angles are as the ſides, by the fifth hereof.



24 *A Triangle of ſome equall ſides, is either equicrurall or equilaterall.*

25 *An equicrurall triangle, is that which hath onely 2. equal ſides.*

26 *An equicrurall triangle, is equiangled at the baſe and contrary* by the fifth hereof.

27 *An equilaterall triangle, (ſo called through the excellency thereof) is that, that hath all 3. ſides, equall one to another.*

28 *An equilaterall triangle is equiangled, and contra.* by the fifth hereof.

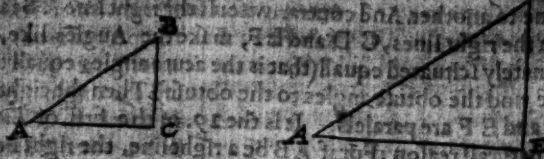
29 *Moreouer a Triangle is right angled, or oblique angled.*

30 *A right angled triangle is that, that hath one right angle.*

31 *In right angled triangles, hauing onely one right angle, the ſubtendent to the right angle, is commonly called the Hipothenuſa: but the ſides including the right angle, are called the perpendiculer, and the Baſe (or meaſure),* as in the Triangles A B C and D E F the ſides A B and D E are the *Hipothenuſas*; B C and E F the *Perpendiculers*; A C and D F the Baſes: or contrarilie, A C and D F are

the

the perpendiculers, and B C. and E F. are the bases.



32 *An oblique angled triangle, is that which hath all the angles oblique.*

33 *An oblique angled triangle, is either obtuse angled, or acute angled.*

34 *An obtuse angled triangle is that, that hath but one obtuse angle.*

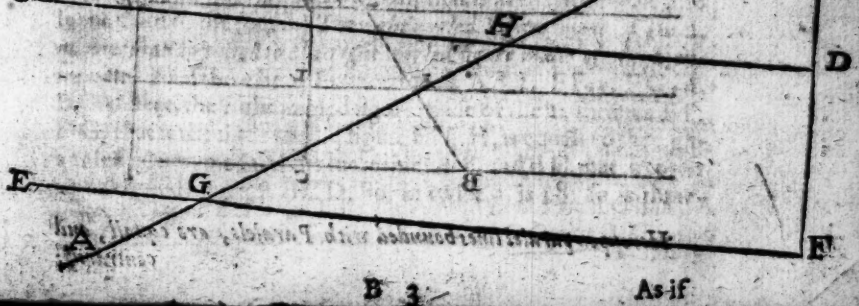
35 *An acute angled triangle, is that that hath all the 3 angles acute.*

36 *Lastly, a triangle is either plaine or Spherical: plaine is a plaine, and Spherical upon the Globe.*

37 *The sides of a plaine triangle in Trigonometria, are right lines only.*

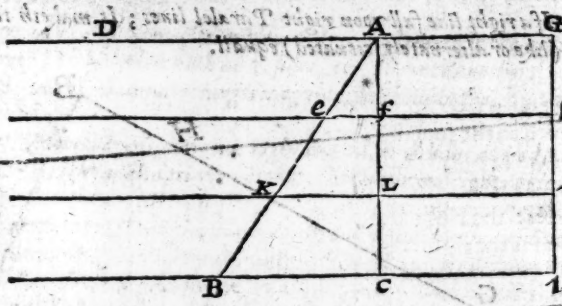
Touching right lines, for the better vnderstanding of Trigonometria: It is necessarie to knowe these Theorems following.

38 *If a right line fall vpon right Paralel lines; It maketh the angles (like or alternately situated) equall.*



As if the right line A B fall vpon the paralels, C D and E F, the angles likely ſcituated, are B H D and B G F. Alſo the angles alternately ſcituated, are C H G. and H G F &c. which are equall all one to another. And contrariwiſe: If the right line A B falling vpon the right lines, C D and E F, make the Angles like, and alternately ſcituated equall (that is the acute angles equall to the acute, and the obtuſe angles to the obtuſe) Then the right lines C D and E F are paralels. It is the 29. of the firſt of *Euclide*. The naturall reaſon: For if A B be a right line, the right lines C D and E F cannot be equally diſtant one from an other, vnleſſe they incline to the right line A B with equall angles. *From hence may be gathered, If many right lines be perpendiculer to one right line, they are paralel one to an other.* As the right lines C D and E F are paralel one to another, becauſe they are perpendiculer to the one, and the ſame line D F.

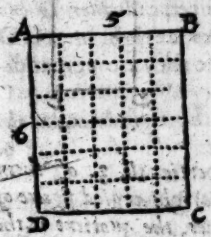
39 *If many right lines, are cut by diuers other right lines, paralel one to an other; the interſegments are proportionall.* As if the 2. right lines A B and A C are cut by the paralels, E H. K M, and B I. I ſay the interſegments A E and A F and likewiſe E B and F C. are proportionall one to another: that is to ſay, If A E. be the ; part of the right line A B. A F. alſo ſhall be ; part of the right line A C. &c. The reaſon is, becauſe the right line E H cutteth of ; part from the whole ſpace D G I B. And therefore from euery line drawne in that ſpace.

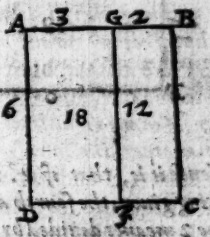


Here vpon paralel lines bounded with Paralels, are equall, and contrarily:

contrarily: As the paralels A F and G H bounded with the paralels A G and F H are equall: For ſithence the whole lines A C and G I are equall, alſo of neceſſitie A F and G H being ½ part thereof, are alſo equall.

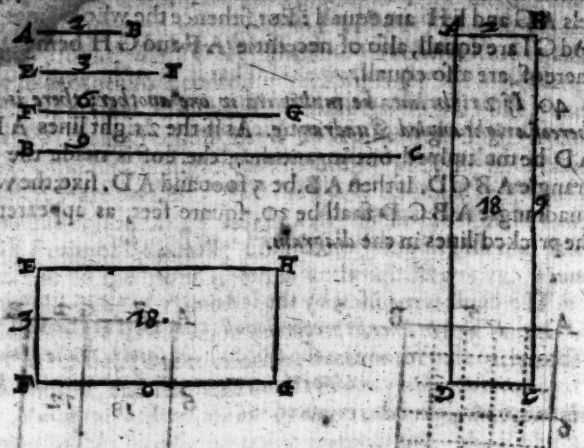
40 *If 2 right lines be multiplied in one another; there is made thereof a right angled Quadrangle.* As if the 2 right lines A B and A D be multiplyed one in another; thereof is made the quadrangle A B C D. If then A B. be 5 foot and A D. ſixe; the whole quadrangle A B C D ſhall be 30. ſquare feet, as appeareth by the pricked lines in the *diagram.*





41 *The right angled figures made of one of the whole lines of one ſide of that figure, and the ſegments of the other ſide thereof added together, are equall to the right angled figure made of both the whole lines.* As the right angled figures made of the whole A D 6, and the ſegments A G 3. and G B 2. that is the right angles A G F D. 18. and F G B C. 12, added together, are equall to the right angled figure A B C D 30. made of both the whole lines A D. 6. and A B. 5.

42 *If 4 right lines are proportionall* (that is as the firſt to the 2. ſo is the third to the fourth) *The right angled Figure made of the 2. meanes ſhall be equall to the right angled figure made of the 2 extreames.* As if there be 4 Proportionals A B. 2. E F. 3. F G. 6. B C. 9. feet, the right angled figure made of the 2. meanes E F. F G, that is the right angled figure E F G H, is equall to the right angled figure, made of the extreames A B. and B C. that is to the right angled figure, A B C D. For as twice 9. is 18: ſo is three-times 6, 18.



1 *Hence it is, that if 4. right lines be proportionall, 3. of them being giuen, the fourth also is giuen: for the right angled figure of the 2 meanes deuided by one of the extreames, the quotient is the other extreame*: As if it were said, As 2 to 3. So is 6 to 9.

The right angled figure made of 3 and 6, that is 18, deuided by the first extreame 2, the quotient is the last extreame 9, &c. And this is the reason why in the Rule of proportion commonly called the Rule of 3. the 2 latter tearmes are multiplyed together, and that product diuided by the first; viz. because the product of the Multiplication of the second, and third tearmes, is also the multiplycation of the first and fourth tearmes, which diuided by the first, shewes the fourth. For diuision and multiplication produce one another mutually: And it is nothing materiall in the order of worke whether of the 2 meanes you put in the second or third place. For either you may say.

As 2 to 3. so 6 to -&c. Or
As 2 to 6 so 3 to &c.

Although the proportion of the first, to the second, and of the third, to the fourth, be one in the former, and another in the latter placing of the tearmes, yet you shall finde the same

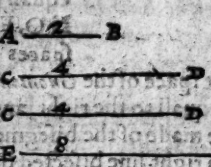
ſame anſwere in both; becauſe it is all one whether you multiplye 3 by 6, or 6 by 3 &c.

2 *Hence alſo it is, that equall right angled figures haue their ſides reciprocally proportionall.* That is in right angled figures, as the leſſer ſide of the firſt right angled figure, to the leſſer ſide of the ſecond right angled figure: So is the greater ſide of the ſecond, to the greater ſide of the firſt right angled figure. *And Contra:* As in the equi-rectangled figures A B C D and E F G H appeareth.

As A B. to E F. So is F G. to B C. &c.
2 3 6 9

The cauſe is manifeſt by the laſt *diagrams* afore going

43 *If 3 right lines be proportionall* (that is, if as the firſt to the ſecond, ſo that ſecond to the third:) *the ſquare made of the mean is equall to the oblong made of the extreames.* For that the Meane is twice put, after this manner.



It is all one as if they were 4. proportionalls: Therefore whatſoeuer hath bene ſaid of 4. proportionalls, wee are alſo to vnderſtand of 3. proportionalls.

44 *If a right line being cut into 2. equall parts, be continued out at length; An oblong made of the line continued, and the line of continuation, is equall to a ſquare made of a right line (of one of the biſegments, and the line of continuation added together) leſſe by the ſquare of the ſaid biſegment by the 6. P:* 2 Euclide

Let A K. be a right line cut into 2. equall parts in the point G. and continued to the point B. And let B C. be equall to the continuation K B. And thereof let be made the oblong A B C D.

Moreouer, let the ſquare G B E F. be made of the right line

 G B. which



G B. which is equall to one of the bisegments G K. & the line of continuation K B. added together: from which square (by the right lines K L. and C H) let the square of the bisegment I L F H. be cut off, that the *Gnomon* M N O. may remaine.

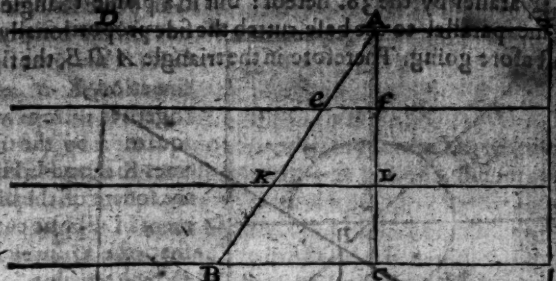
I say that the oblong A B C D. is equall to the square G B E F. lesse by the square I L F H. or which is all one: I say the oblong A B C D. is equall to the *Gnomon* M N O. For the figures, or spaces M. and N. are common to both: But the space of the *Gnomon* O. or the right angled figure I C E L. is equall to the right angled figure G H D A. For both of them are made of the bisegment and the continuation. Therefore if a right line bisected be continued &c. which was to be demonstrated.

And thus much of right lines, as of the sides of plaine triangles, I haue thought good to set downe. Now I will returne to plaine triangles themselues.

45 *In a plaine triangle, a line drawne parallel to the base, cutteth the sides thereof proportionally.*

As in the plaine triangle A B C. If K L. be parallel to the base B C. it cutteth off from the side A C. ⅓ part, and also it cutteth from the side A B. ⅓ part by the 39. hereof: And so they shall be proportionall.

As A B. to A C. so is A K. to A L. Also,
As A K. to B K. so is A L. to C L. Also,
As A K. to A L. so is K B. to L C.



*46 If diuers plaine triangles are compared together * Equiangled triangles haue their sides about the equall angles proportionall, and Contra: by the* 4. *P.* 6 Euclide.

This Theorem is the chiefe ground of *Trigonometria*. Therefore aboue others, it is to be diligently explaned & noted.



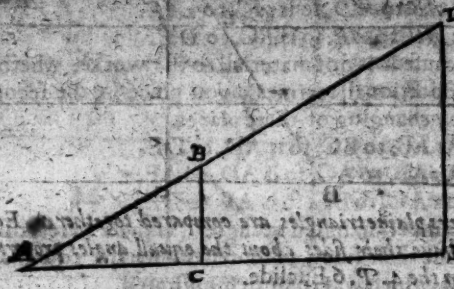
The declaration: Let *A B C.* and *A D E.* be 2. plaine equiangled triangles, so as the angles at *B.* and *D.* at *A.* & *A.* and also at *C.* and *E.* be equall one to another: I say their sides about the equall angles are equall; that is

1. *As A B.* to *B C.* so is *A D.* to *D E.*
2. *As A B.* to *A C*, so is *A D.* to *A E.*
3. *As A C.* to *C B.* so is *A E.* to *E D.*

The Demonstration. For, because the angles *B A C.* and *D A E*, are equall, by the *Pro:* therefore if *A B.* be applyed to *A D. A C.* shall of necessity fall in *A E.* and by such applycation, such a figure shall be made.

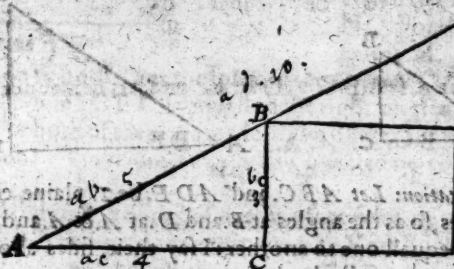
In which, figure, because that *A B.* and *A C.* doe meet together, and also the angles at *B.* and *D.* and at *C* and *E.* are equall; by the *Pro:* therefore th'other sides *B C.* and *D E*, shal be of ne-

ceſſitie parallel by the 38. hereof: but in a plaine triangle, a right line parallel to the baſe, cutteth the ſide proportionally by the laſt afore going. Therefore in the triangle *A D E*, the right



line *B C*, being parallel to the baſe *D E*, cutteth the ſides *A D*, and *A E*, proportionally: and ſo

As A B. to A D. ſo is A C. to A E.



Moreouer by the point B. let the right line B F. be drawne parallel to the baſe A E. and it ſhall cut the other 2. ſides D E. and D A. proportionally in the pointes B. and F. by the ſame laſt afore going. And then the ptoportion ſhall be

As AB. to AD. ſo is FE to DE. or which is all one
As AB. to AD. ſo is BC. to DE.

For F E. and BC. are equall by the 39. hereof: Beſides ſithence they are:

As AB. to AD. ſo AC. to AE.
And ſo BC. to DE they ſhall be alſo
As AC. to AE. ſo BC. to DE.

For

For what things are agreeable to one third, are agreeable alſo to one another: Therefore generally

1. As *AB.* to *AD.* ſo is *BC.* to *DE.*
2. As *AB.* to *AD.* ſo is *AC.* to *AE.*
3. As *AC.* to *AE.* ſo is *BC.* to *DE.*

Laſtly, becauſe it is not materiall to the worke, whether of the meane proportionall tearmes, you place in the ſecond or third place, by changing of theſe places.

1. As AB. to BC. ſo is AD. to DE.
2. As AB. to AC. ſo is AD. to AE.
3. As AC. to BC. ſo is AE. to DE.

And ſo plaine equiangled triangles (as theſe here A B C. & ADE. are) haue their ſides, comprehending the equall angles proportionall, which was to be demonſtrated.

The illuſtration by numbers: Let AB. be 5. feet: AD. 10. DE. 06. and it is demaunded how many feet is BC.

As AD. 10. to DE. 06. ſo is AB. 05.

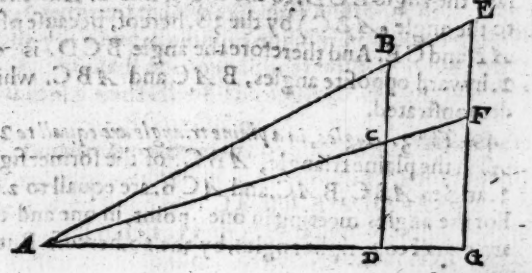
$\frac{30}{10}$ (to 3. BC.

Let A C. be 4. feet, B C. 3. D E. 6. and it is demaunded how many feet is A E.

As BC. 3. to CA. 4. ſo is DE. 6.

$\frac{24}{3}$. (to 8. AE.

47. *If diuers plaine Triangles be compounded, and be cut with right lines paralels, the interſegments are proportionall,* As: for example. If the 2. Triangles E A F & F A G. be compounded, and be cut with the right paralel lines B C D. and E F G. their



interſegments are proportionall.

As B C. to E F. ſo is, C D. to F G. Or

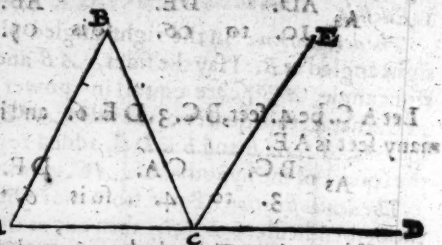
As B C. to C D. ſo is, E F. to F G. &c. by the 39. hereof, or by the laſt precedent: For the triangles A B C. and A E F. are equiangled by the 38. hereof, becauſe B C. and E F. are paralels: therefore

As A C to A F ſo is, B C to E F by the laſt afore going: but by the ſame,

As A C. to A F. ſo is, C D. to F G. & thoſe that are agreeable to a third, are alſo agreeable to one another: therefore they are alſo,

As B C. to E F. ſo is C D. to F G. &c.

48 *If any ſide whatſoever, of a plaine triangle be continued, the outward angle, made by that continuation, is equall to the 2. inward oppoſite angles.* As, if in the plaine triangle, A B C. the ſide A C be continued to D. the outward angle, B C D. ſhall be equall to the 2. inward oppoſite angles, B A C and A B C. For if from the point, C. were drawne the right line C E paralel to the right line, A B. the outward angle, B C D. ſhall be compounded of the angles, A E C D, and E C B. But the angles, E C D and E C B. are equall to the 2. inward oppoſite angles, B A C and A B C. (that is to ſay, the angle E C D. to the angle, B A C. And the angle B C E to the angle A B C) by the 38. hereof, becauſe of the paralels, A B and C E. And therefore the angle B C D. is equall, to the 2. inward oppoſite angles, B A C and A B C. which was to be demonſtrated.



49 *The 3. Angles, in a plaine triangle are equall to 2. right angles.* As in the plaine triangle, A B C. of the former figure: I ſay, the 3. angles A B C, B A C, and A C B. are equall to 2. right angles. For the angles meeting in one point, in one and the ſame line, are equall to 2. right angles, by the 20. hereof. But the 3. angles

A B C.

ABC. BCA. and *BAC.* are equall to the 3. angles, meeting in the point, C. vpon the ſame line *AD.* For the angle *BCA.* is common to both, and the angles *ECD.* and *ECB.* are equall to the angles, *BAC* and *ABC.* by the laſt afore going. Therefore the 3. angles, *ABC. BCA.* and *BAC.* are equall to 2. right angles, which was to be demonſtrated. Hence is it, that

1 *In a plaine triangle, there cannot be but one right, or one obtuſe angle.*

2 *And one angle being right or obtuſe, the other 2. are neceſſarily acute angles.*

3 *And the third angle, is the complement of any of the other 2. to two right angles.*

4 *And laſtly, if 2. triangles are equiangled, in 2. of their angles, they are wholly equiangled.*

5o *In a right angled plaine triangle, the ſides including the right angle, are equall in power, to the hypothenuſa.* By the laſt Pro: 1. *Euclide.*

The declaration: In the right angled plaine triangle, *ABC.* right angled at *B.* I ſay the ſides, *AB* and *BC.* including the right angle, *ABC.* are equall in power to the hypothenuſa *AC.* that is, the ſquares of the ſides, *AB* and *BC.* to wit, the ſquares, *ALMB* and *BEDC.* added together, are equall to the ſquare of the hypothenuſa, *AC.* to wit, the ſquare *ACKI.*

The demonſtration: For if from the right angle, *B.* be let fall the perpendiculer, *BFG.* then out of the ſquare, *ACKI.* is made the 2. oblougs *AFGI* and *FCKG.* which are equall, to the ſquare, *BEDC.* & that other to the ſquare *ALMB.* And therefore the ſquare, *ACKI.* compounded of thoſe 2. oblougs, is equall to the 2. ſquares, *ALMB* and *BECD.*

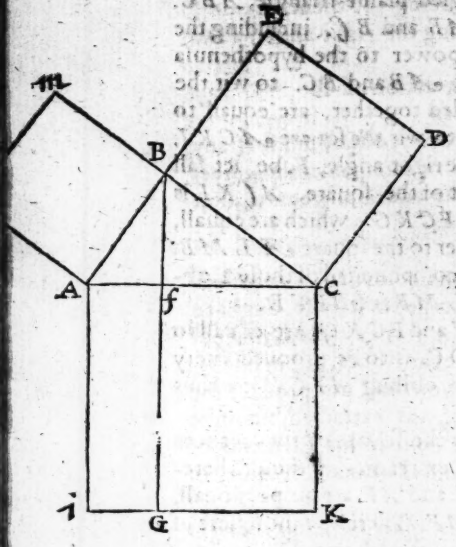
But that the 2. oblougs, *AFGI* and *FCKG.* are equall to the 2. ſquares, *ALMB* and *BEDC.* is to be prooued euery one in particuler. And firſt of the obloug *AFGI.* it is thus prooued.

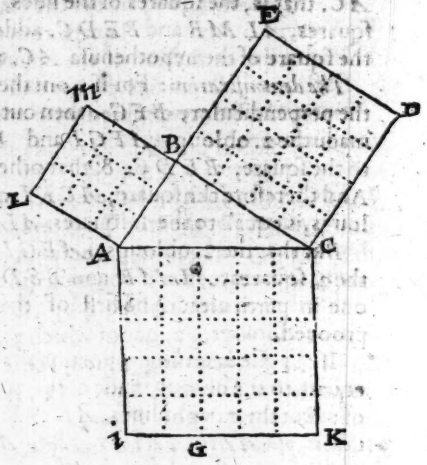
If 3. right lines be proportionall, the ſquare of the meane is equall, to the obloug made of the 2. extreames by the 43. hereof: But the 3. right lines, *AI. AB.* and *AF.* are proportionall, that is, as *AI* to *AB* ſo is *AB* to *AF.* Therefore the ſquare of *AB*, is equall to the oblouge, made of *AI.* and *AF.*

The *Minor* is prooued, for the triangles, *A B C.* and *B A F.* are equiangled, becauſe of the common angle, at *A.* and the 2. right angles, *B.* and *F.* by the 4. conſectarie of the 43. hereof: therefore by the 46. hereof, as *A C.* (equall to *A I.*) to *A B.* ſo is *A B* to *A F.*

In like manner it is altogether prooued, that the oblong *F C K G.* is equall to the ſquare, *B E D C.* For the triangle *A B C* and *B C F.* are equiangled, becauſe of their common angle, at *C.* and the 2. right angles, at *B.* and *F.* by the 4. conſectarie of the 49. hereof.

Therefore by the 46. as *A C.* to *B C.* ſo is *B C.* to *F C.* And ſo by the 43. hereof, the ſquare of *B C.* is equall to the oblong, made of the lines, *A C.* or *K C.* and *F C.* Therefore in a right angled plaine triangle, the ſides including the right angle, are equall in power, to the hypothenuſa, which was to be demonſtrated.





By

Commentarie.

By a more mechanicall way, this Proe may be demonſtrated, viz. Let *A B C.* be a triangle, right angled at *B.* and let *A B.* be 3. *B C.* 4 and *A C.* 5. feet: let euery ſide be ſquared, and let euery ſquare, be diſtinguiſhed into ſquare feet, by the pricked lines: you ſhall ſee the ſquare of the hypothenuſa, *A C.* to haue in it ſo many ſquare feete, as the ſquares of *A B.* and *B C.* taken together.

Conſectarie.

Therefore in a right angled plaine triangle, any of the 2 ſides being giuen, the third may be ſaid to be giuen. As if the 2 ſides including the right angle, *A B* and *B C.* be giuen: viz. 3. and 4. their ſquares 9. and 16. being added together, is 25. the ſquare roote thereof being extracted, the hipothenuſa *A C.* ſhall be found 5. parts

Contrarilie, if he hipothenuſa 5. and one of the ſides, including the right angle 3. be giuen, Subſtract the ſquare, of 3. from the ſquare of 5. that is, the ſquare 9. being ſubtracted from the ſquare, 25. and out of the remainder, being 16. the ſquare root being extracted: the other ſide including the right angle, ſhall be found, 4. parts.

Commentaries, about the extraction of the ſquare roote.

1 If after the extraction of the ſquare root of any number any fractions ſhall remaine, put for denominater vnder thoſe fractions, the Roote doubled with one added thereunto, after this manner.

$$\overset{3}{\cdot} \quad x\alpha(3\tfrac{1}{7}.$$

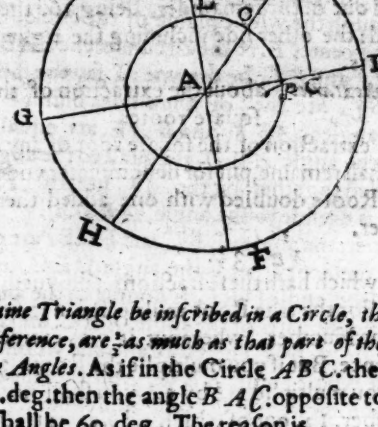
2 The Root which hath theſe fractions adioyned, is neuer exactly true. For the true Root multiplied in it ſelfe, ought to produce the Number, whereout it was extracted, without any difference. But if you multiplie the Roote, $3.\frac{1}{7}$ in it ſelfe, that is, if you multiplie $3.\frac{1}{7}$ by $3.\frac{1}{7}$ you ſhall not produce the true Number, 12. out of which $3.\frac{3}{7}$ was extracted, but onlie $11\frac{37}{49}$. Concerning which, ſee *Ramus* in his Elements of *Geometrie*. Elem. 8. Lib. 12. And *Lazarus Schoner*, in his comment vpon *Ramus*.

51 *In a plaine right angled triangle, the ſides including the right*

angle, are for the moſt part irrationall to the hipotheuuſa: that is, inexplicable in an exact number, of what quantitie ſoeuer. The cauſe appeareth by the ſecond commentarie next before going.

52 *In a plaine right angled triangle, the one of the acute angles, is the complement of the other,* by the 49. hereof, It is very eaſilie prooued in this manner.

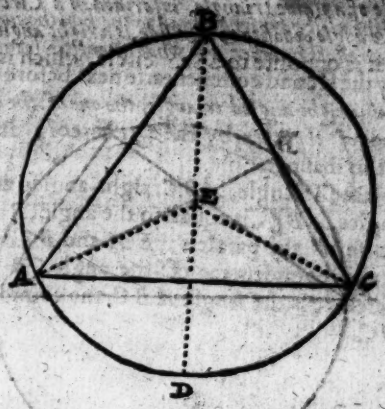
In the plaine Triangle, *A B C.* right-angled at *C.* the one of the acute angles, *A B C.* is equall to the angle, *B A E,* by the 8. hereof becauſe of the paralels, *E A* and *B C.* But the angle *E A B.* is the complement of the angle, *B A C.* by the worke; Therefore is the Angle *A B C.* the complement of the angle, *B A C.*



53 *If a plaine Triangle be inſcribed in a Circle, the angles oppoſite to the circumference, are ½ as much as that part of the circumference oppoſite to the Angles.* As if in the Circle *A B C.* the circumference *B C.* be 120. deg. then the angle *B A C.* oppoſite to the circumference *B C.* ſhall be 60. deg. The reaſon is.

Becauſe the whole circumference *A B C.* is 360. degrees by the 7. hereof. And the 3. angles of the Triangle, *A B C.* inſcribed in the Circle, is 180. de. by the 49. hereof: Therefore as euery arch, is the ⅓ part of 360. de. ſo euery angle oppoſite to that arch, is the ⅓ part of 180. deg.

It is



It is more plainly thus demonſtrated: As for example, of the Angle *A B C*. From the ſaid angle, *A B C*. let the diameter *B E D*. be drawne through the whole plaine of the circle.

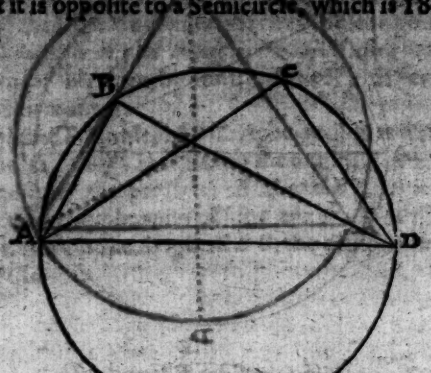
And from the center *E*. to the circumference *A B C D*. let the two *Rudij E A*. and *E C*. be drawne: I ſay the diuided angles, *A B D* and *D B C*. are the ½ of the angles diuided *A E D*. and *B E C*. For the angles, *A B E*. and *B A E*. are equall by the fifth thereof: And the angle *A E D*. is equall to the angles, *A B E*. and *B A E*. added together by the 48. hereof. Therefore the angle *A E D*. is double to the angle, *A B D*.

In like manner, the angles E B C and E C B. are equall by the 5. and to both theſe together, is the angle D E C. equall, by the 48. Therefore the angle D E C. is double to the angle, D B C.

Then becauſe the parts of the angle, A E C. are double to the parts of the angle, A B C. Therefore alſo the whole angle, A E C. is double to the whole angle, A B C. And thereupon the angle A B C. is ½ of the angle, A E C. and conſequently ½ of the arch, A D C. which is the meaſure of the angle A E C. The ſame prooſe is of the reſt. If therefore a plaine triangle be inſcribed in a circle, the angles oppoſite to the circumference are ½ of that part of the circumference oppoſite to the angles which was to be demonſtrated. *Hence it is, that*

1 *If the*

1 *If the side of a plaine triangle, inscribed in a Circle, be the Diameter: the angle opposite to that side, is a right angle:* that is 90. d. for that it is opposite to a Semicircle, which is 180. d.

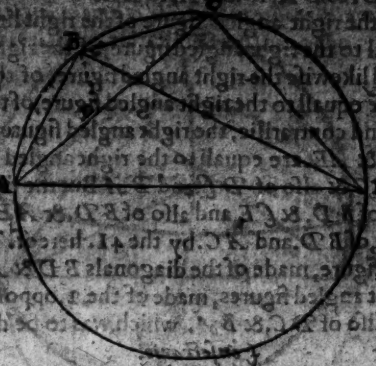


2 *If vnlike triangles right lined, be inscribed in the same segment of a Circle, vpon one base: the Angles in the circumference are equall.* As the two Triangles, A B D. and A C D. being inscribed in the same segment of the circle, A B C D. vpon the same Base A D. are equiangled in the points, B. and C. falling in the circumference: For the same Arch, A D. is opposite to both those angles, that is to the Angle; A C D. and also to the angle A B D.

3 *If 2. plaine triangles inscribed in the same segment of a Circle, vpon the same base, be so ioyned together in the toppe, (or in the angles, falling in the circumference) that thereof is made a quidrilateral figure, intersected with diagonals; The right angled figure, made of the diagonals is equall to the right angled figures (added together) made of the opposite sides.* Ptolomy *and* Copernicus.

The declaration: Let A B D. and A C D. be 2. Triangles, inscribed in the same segment of the circle, A B C D. vpon the same base A D. so ioyned in the top by the right line, B C. that thereof is made the 4. sided figure, A B C D. I say that the right-angled figure made of the 2. diagonals, A C. and B D. is equall to the right-angled figures together, made of the opposite sides,

A B &



A B. and D C. and also of the sides, B C and A D.

The demonstration. For if at the point B, you make the angle, A B E. equall to the angle, D B C. and so you cut the diagonall, A C. into 2. partes by the right line, B E. At the point E. it is manifest that the right angled figures of B D and E C. and also of B D and E A. are equall to the right angled figures, made of B C and D A. and also of C D, and A B. For if 4. right lines be proportionall, the right angled figure made of the meanes, is equall to the right angled figure made of the extreames by the 42. hereof. But the 4. right lines, B D. D A. B C. and C E. are proportionall. For because the triangles, A B D and B C E. are equiangled, because of the equall angles, B C A. and B D A. by the 2. consect. aforegoing: also because of the equall angles, A *B* D & E *B* C. (which are equall, for that the same, E *B* D. is added to the equall angles, A B E and D B C. and lastlie, because of the equall angles, *B* E C. & *B* A D. by the 4. consectarie 49. Therfore their sides are; As *B* D. to D A. so is *B* C to C E. In like manner the 4. right lines, *B* D. D C. *B A*. & A E. are proportionall.

For because the Triangles, *B* D C & *B* A E. are equiangled because of their equall angles, *B* D C & *B* A E, by the 2. consect arie aforegoing. Also because of their equall angles, D *B* C. & A *B* E. by the proposition; & lastly because of the equall angles, *B* C D & *B* E A. by the 4. consectarie of the 49. Therefore their

sides

ſides are as B D. to D A. ſo is B A. to A E.

Therefore the right angled figure of the right lines, D A. and B C. are equall to the right angled figure, of the right lines, B D. and C E. And likewiſe the right angled figure, of the right lines, D C & B A. are equall to the right angled figure, of the right lines B D & A E. And contrarilie, the right angled figures, B D & C E. and alſo *B D* & *A E*. are equall to the right angled figures, made of *D A* and *B C* & alſo of *D C*. and *B A*. But the right angled figures, made of *B D*. & *C E*. and alſo of *B D*. & *A E* are the right angled figure, of *B D*. and *A C*. by the 41. hereof. Therefore the right angled figure, made of the diagonals *B D* & *A C*. is equall to the 2. right angled figures, made of the 2. oppoſite ſides *D A*, and *B C*. & alſo of *D C*. & *B A*. which was to be demonſtrated.

Conſectarie.

Therefore in a quadrilater all figure inſcribed in a Circle, and interſected with diagonalls, and ſo conſiſting of 6. right lines. And 5. of them being giuen, the 6. is alſo giuen. You haue moſt excellent examples hereof in the ſecond booke. *Pro:* 32. 33. 35. 36. 37. 38.

And thus much of plaine Triangles.

It followeth of Sphæricall.

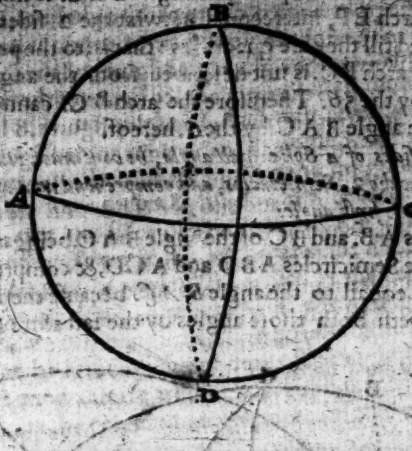
55. *The ſides of a Spharicall triangle, are the arches of great Circles, euery one being leſſe then a ſemicircle.*

56. *A great circle of the Sphere is that which deuideth the whole Sphære into 2. hemiſpheres, and ſo is euery where diſtant from his Poles by a Quadrant of a great circle.*

57. *If a great circle of the Sphære, paſſe by the poles of another great circle, they cut one another at right angles: and Contra.*

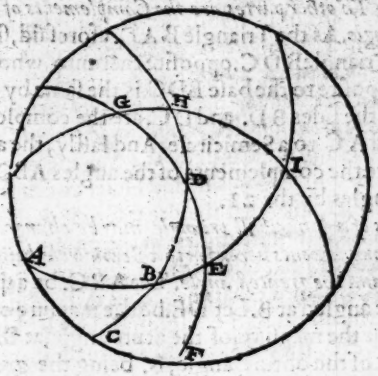
Let A E C. be a great Circle of the Sphære, whoſe poles let be B. and D. by which poles B. and D. let another great Circle paſſe, being B E D, I ſay that the great Circle *B E D*. cutteth the great Circle A E C. at right angles, at the points E. and F. For vpon the pole E. or F. let alſo another great circle A *B C D*. be deſcribed, It is manifeſt that the arches A B. B C. C D. and D A. ſhall be the meaſures of the angles E. and F. by the 6. But that the arches A B. B C. C D. D A. are quadrants, by the laſt

aforegoing:



aforegoing: Therefore the angles at E. and F. are right angles by the 15. which was to be demonstrated.

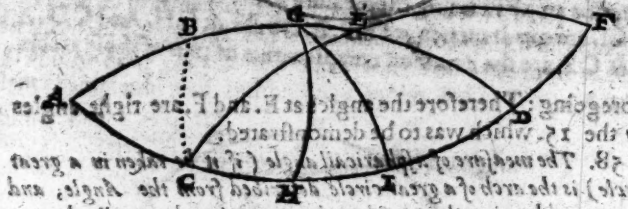
58. *The measure of a spharicall angle (if it be taken in a great circle) is the arch of a great circle described from the Angle, and intercepted betwixt the 2. sides, being continued out till they are Quadrants*, by the 6. and 56. hereof.



As the meaſure of the Spherical angle B A C. is not the arch B C. but the arch E F, intercepted betwixt the 2. ſides A B. and B C. continued till they are quadrants: that is; to the points E. & F. becauſe the arch B C. is not deſcribed from the angle A. but the arch E F. by the 56. Therefore the arch B C. cannot be the meaſure of the angle B A C. by the 6. hereof.

59 *If the ſides of a Spherical angle, be continued till they meet together, they make 2. Semicircles, and comprehend an angle equall, and oppoſite to the firſt angle:*

As the ſides A B. and B C. of the angle B A C. being continued to D. make the Semicircles A B D. and A C D. & comprehend the angle B D C. equall to the angle *B A C.* becauſe the ſame arch G H. meaſureth both thoſe angles by the laſt afore going.

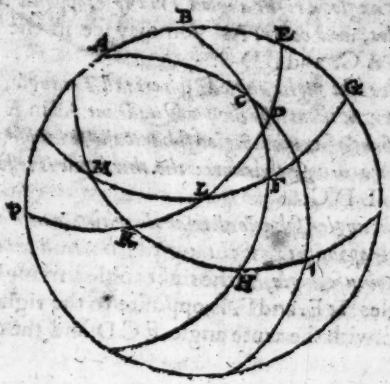


60 *Euery Spherical Triangle, from euery angle hath another triangle oppoſite thereunto, whoſe baſe and the angle oppoſite to the baſe, are the ſame: Th'other partes are the Complements of the parts of the* former *Triangle.* As the Triangle B A C. aforeſaid, from the angle A, hath the Triangle B D C. oppoſite thereunto, whoſe baſe B C, & the angle oppoſite to the baſe B D C. is the ſame by the laſt aforegoing: and the ſides B D. and D C. are the complements of the ſides A B. and A C. to a Semicircle. And laſtly, the angles D B C. and D C B. are the complements of the angles A B C. and B C A. to two right angles by the 21.

61 *The ſides of a ſpherical triangle, may be changed into angles & contra: the complements hereupon to a Semicircle, being taken for the greateſt ſide, and the greateſt angle.* Let A B C. be a ſpherical Triangle, obtuſe angled at B. Let D E. be the meaſure of the angle, at A. Let F G. be the meaſure of the acute angle, at B. (which is the complement of the obtuſe angle, B. being the greateſt angle in

the

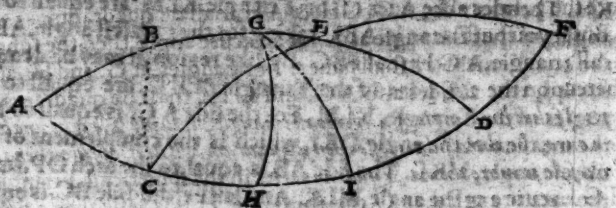
the giuen Triangle,) and let H I. be the meaſure of the angle, at C. K L. is equal to the arche, D E. becauſe K D. & L E. are quadrants, and their common complement is, L D. L M. is equall to the earch F G. becauſe L G. and F M. are quadrants, & their common complement is, L F. K M. is equall to the arch H I. becauſe K I. and M H. are quadrants, and their common complement is K H. Therefore the ſides of the Triangle, K L M. are equall to the angles of the Triangle, A B C. taking for the greateſt angle, A B C. the complement thereof F B G. By like reaſon, it may be demonſtrated, that the ſides of the Triangle, A B C. are equall to the angles of the Triangle, K L M. For the ſide A C. is equall to D I. the meaſure of the angle, D K I. which is the complement of the obtuſe angle, M K L. The ſide A B. is equal, to the arch O P. being the meaſure of the angle, M L K. And laſtlie, the ſide B C. is equall to the arch F H. being the meaſure of the angle, L M K. For A D. & C I. are quadrants: ſo are A P. & O B. F. & C H. And C D. A O. & C F. are the common complements of two of thoſe arches.



Therefore the ſides of a ſphæricall Triangle, may be changed into Angles, and contr: which was to be demonſtrated.

62 *A right angled ſphericall Triangle, hath one right angle or more then one.*

63 *One right angle with 2. acute angles, as B A C. or with 2. obtuse angles, as B D C. or with one obtuse and one acute angle, as C D E.*
For I suppose the angles, at A. and D. to be right angles.



64 *A right angled spharicall Triangle, with 2. acute angles, hath from the right angle, a right angled Triangle, opposite thereunto, with 2. obtuse angles, and contr:* As you may see in the right angled Triangles, B A C. and B D C.

65 *The sides of a right angled spharicall Triangle, with 2. acute angles, are euery of them lesse then a Quadrant.* As in A B C.

66 *The 2. sides of a right angled spharicall Triangle, with 2. obtuse angles, are more then Quadrants; the third side is lesse then a Quadrant,* As in B D C.

67 *A right angled spharicall triangle, with 2. acute angles, is from the acute angle opposite to a right angled spharicall triangle, with one acute and one obtuse angle,* As the right angled triangle EDF. with 2. acute angles, at E. and F. is opposite to the right angled Triangle, C D E. with the acute angle, E C D. and the obtuse angle, C E D.

68 *The sides subtending the right angles of a spharicall Triangle, having diuers right angles, are Quadrants.*
The reason is, for that, (as in the Triangle, AGH.) If the great Circles, A G. & A H. do cut the great circle, G H. at right angles in the points G. & H. A is the pole of the great circle. G H. by the 57. And A G. & A H. are quadrants by the 56. But if the angle

angle at A. be alſo a right angle, then G H. is alſo a quadrant by the 5 8. and 15. hereof.

69 *A ſpharicall Triangle, hauing diuers right angles, hath either 3. or 2. right angles: And ſo of the ſides, hath 3. or 2. quadrants.* As if you put the angle, at A. for a right angle, the ſphæricall Triangle, A G H. ſhall haue 3. right angles, at A. G. and H. and therefore the 3. ſides alſo, A G. G H. & A H. ſhall be quadrants.

But if you put the angle at A. for an acute angle, then the ſphæricall triangle, A G H. ſhall haue 2 right angles, at G. and H. and thereupon the 2. ſides alſo A G. and G H. ſhall be quadrants.

70 *If the third angle of a ſpericall triangle, hauing 2. right angles, be acute, the third ſide is leſſe then a quadrant. But if obtuſe, then the third ſide is more then a Quadrant.* As in the ſphæricall Triangle, H G I. acute angled, at G. the third ſide, H I. is leſſe then a quadrant. In the ſphæricall Triangle, A G I. obtuſe angled, at G. the third ſide A I. is more then a Quadrant.

The former Diagram ſheweth the Demonſtration hereof.

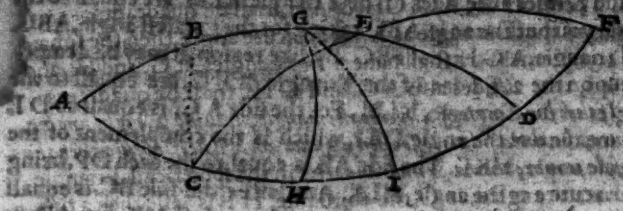
71 *An oblique ſpharicall Triangle, conſiſteth ſimplie of acute angles, or obtuſe angles, or of both of them mixed together.*

72 *A ſpharicall triangle, with 2. obtuſe angles, and one acute angle, is oppoſite to a ſpharicall triangle, ſimply acute angled. And contra:* As if the angles, at A. and D. be ſuppoſed acute, then the triangle *B D C.* with 2. obtuſe angles, at B. and C. and one acute angle at D. is oppoſite to the ſimplie acute angled triangle, *A B C.*

73 *A ſpharicall triangle, with 2. acute angles, and one obtuſe angle, is oppoſite to a triangle ſpharicall, ſimply obtuſe angled: and contra:* As if the angles, at A. & D. be ſuppoſed obtuſe, then the triangle, *A B C.* with 2. acute angles, at *B.* & *C.* and one obtuſe angle, at *A.* is oppoſite to the ſimply obtuſe angled triangle, *B D C.*

74 *The 3. Angles of euery ſpharicall Triangle, are more then two right angles.*

63 *One right angle with 2. acute angles, as B A C. or with 2. obtuse angles, as B D C. or with one obtuse and one acute angle, as C D E.*
For I suppose the angles, at A. and D. to be right angles.



64 *A right angled spheriall Triangle, with 2. acute angles, hath from the right angle, a right angled Triangle, opposite thereunto, with 2. obtuse angles, and contr:* As you may see in the right angled Triangles, B A C. and B D C.

65 *The sides of a right angled spheriall Triangle, with 2. acute angles, are euery of them lesse then a Quadrant.* As in A B C.

66 *The 2. sides of a right angled spheriall Triangle, with 2. obtuse angles, are more then Quadrants; the third side is lesse then a Quadrant,* As in B D C.

67 *A right angled spheriall triangle, with 2. acute angles, is from the acute angle opposite to a right angled spheriall triangle, with one acute and one obtuse angle,* As the right angled triangle EDF. with 2. acute angles, at E. and F. is opposite to the right angled Triangle, C D E. with the acute angle, E C D. and the obtuse angle, C E D.

68 *The sides subtending the right angles of a spheriall Triangle, having diuers right angles, are Quadrants.*
The reason is, for that, (as in the Triangle, A G H.) If the great Circles, A G. & A H. do cut the great circle, G H. at right angles in the points G. & H. A is the pole of the great circle. G H. by the 57. And A G. & A H. are quadrants by the 56. But if the angle

angle at A. be alſo a right angle, then G H. is alſo a quadrant by the 5 8. and 15. hereof.

69 *A ſpharicall Triangle, hauing diuers right angles, hath either 3. or 2. right angles: And ſo of the ſides, hath 3. or 2. quadrants.* As if you put the angle, at A. for a right angle, the ſphæricall Triangle, A G H. ſhall haue 3. right angles, at A. G. and H. and therefore the 3. ſides alſo, A G. G H. & A H. ſhall be quadrants.

But if you put the angle at A. for an acute angle, then the ſphæricall triangle, A G H. ſhall haue 2 right angles, at G. and H. and thereupon the 2. ſides alſo A G. and G H. ſhall be quadrants.

70 *If the third angle of a ſpericall triangle, hauing 2. right angles, be acute, the third ſide is leſſe then a quadrant. But if obtuſe, then the third ſide is more then a Quadrant.* As in the ſphæricall Triangle, H G I. acute angled, at G. the third ſide, H I. is leſſe then a quadrant. In the ſphæricall Triangle, A G I. obtuſe angled, at G. the third ſide A I. is more then a Quadrant.

The former Diagram ſheweth the Demonſtration hereof.

71 *An oblique ſpharicall Triangle, conſiſteth ſimplie of acute angles, or obtuſe angles, or of both of them mixed together.*

72 *A ſpharicall triangle, with 2. obtuſe angles, and one acute angle, is oppoſite to a ſpharicall triangle, ſimply acute angled. And contra:* As if the angles, at A. and D. be ſuppoſed acute, then the triangle *B D C.* with 2. obtuſe angles, at B. and C. and one acute angle at D. is oppoſite to the ſimplie acute angled triangle, *A B C.*

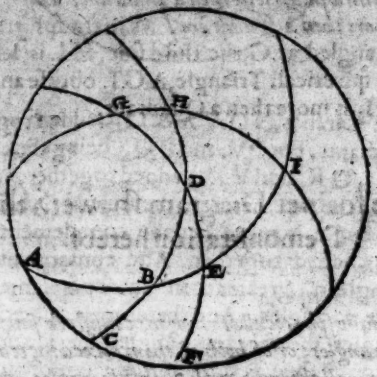
73 *A ſpharicall triangle, with 2. acute angles, and one obtuſe angle, is oppoſite to a triangle ſpharicall, ſimply obtuſe angled: and contra:* As if the angles, at A. & D. be ſuppoſed obtuſe, then the triangle, *A B C.* with 2. acute angles, at *B.* & *C.* and one obtuſe angle, at *A.* is oppoſite to the ſimply obtuſe angled triangle, *B D C.*

74 *The 3. Angles of euery ſpharicall Triangle, are more then two right angles.*

In ſphæricall triangles, hauing more right, or obtuſe angles, then one, whether they be ſimple or compound, the thing is manifeſt of it ſelfe.

In ſphæricall Triangles, of 2. or 3. acute angles, it may be thus demonſtrated.

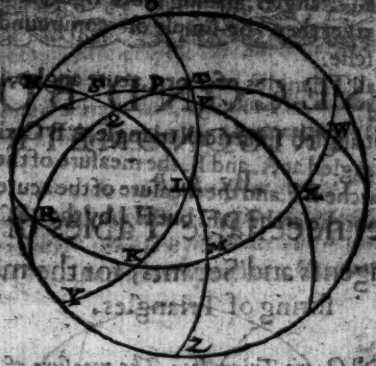
In the right angled ſphæricall triangle, A B C. right angled at C. and acute angled at A. and B. the meaſure of the acute angle, B A C. is the arche E F. and the meaſure of the acute angle, A B C. or D B E. is not the arche D E. but H I. by the 58.



But the arches E F. & D E. are equall to a quadrant. Therefore the arches F E. & H I. added together, are more then a quadrant. And conſequentlie, the angles anſwering to theſe arches, to wit, the angles B A C. & A B C. ioyntlie together, are more then a quadrant, that is, greater then a right angle. But the angle A C B. is a right angle by the Pro: Therefore in the ſphæricall Triangle, A B C. of 2. acute angles, the 3. angles are more then 2. right angles.

In the ſphæricall triangle, K L M. meerely acute angled: the meaſure of the acute angle at L. is the arch N O. the meaſure of the acute angle, at K. is the arch V X. the meaſure of the acute angle, at M. is the arch Q R.

But



But these 3. arches, N O. V X. & Q R. added together, are more then 2. quadrants, For P V. and P Q. (being the complements of the 2. arches, Q R. and V X.) added together, are lesse then the arch, N O. by the pro: therefore the arch, N O. being the measure of the third angle, is more then the complements of the other 2. angles added together. And consequentlie, also the third angle is greater then the complements of the other 2. angles. And therefore in sphæricall Triangles, meerely acute angled, the 3. angles are more then 2. right angles. A more subtill demonstration see in *Regiom.* 4 9. P. 3.

The end of the first booke.

THE SECOND BOOKE OF TRIGONOMETRIA.

By *R. P.*

Of the necessarie Tables of *Sines*, Tangents and Secants, for the measuring of Triangles.

1 SO are *Triangles: The measure of Triangles is the finding out in triangles, the vnknowne sides or angles, by 3. knowne, whether angles, or sides, or both.* It is also called the resoluing of Triangles, or the calculating of Triangles.

There are *Areas* in triangles besides their angles and sides, but the measure of them is not proper to triangles, for we measure the *Area* of any other figures whatsoeuer as well as of them: Neither commeth it first from triangles, but is deriued from Quadrangles to triangles. And therefore appertaineth not to this place.

2 *The dimension of triangles is performed by the golden Rule of Arithmeticke: which teacheth of 4. Numbers proportionall one to another, any 3. of them being giuen, to finde out the fourth.*

3 *Therefore for the measuring of triangles, there must be certain proportions of all the partes of a triangle one to another, and those proportions explained in Numbers.*

4 *The proportions of all the partes of a triangle one to another cannot be certaine, vnlesse euery crooked line in triangles (as in all triangles the measures of the angles are, and in Spharicall triangles also the sides) be reduced to right lines. For of a crooked line, to a crooked line, or to a right line was neuer yet found any proportion, nor perhapps shall euer be.*

5 *Crooked lines are reduced to right lines, by the definition of the quantitie,*

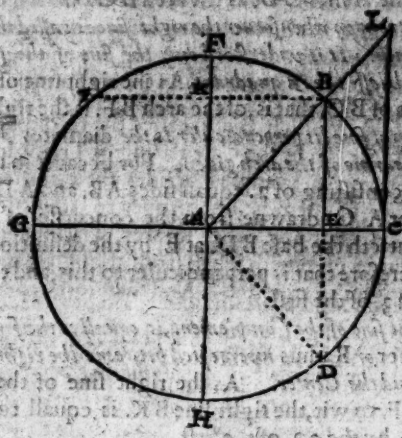
quantitie, which right lines applyed to a Circle have, in respect of the Radius.

6 *Right lines applied to a Circle are* Subtenses, Sines, Tangents, *and* Secants.

7 *A subtense is a right line, inscribed in a Circle, dividing the whole Circle into 2. Segments, and in like manner Subtending both the Segments.*

8 *A Subtense is either the greatest, or not the greatest.*

9 *The greatest Subtense, is that that divideth the whole Circle into 2. equall Segments. And so it subtendeth both the Semicircles as* the right line GC. is commonly called a *Diameter.*



10 *A Subtense not the greatest, is that that divideth the whole Circle into 2. vnequall Segments: And so vn the one side subtendeth an arch lesse then a Semicircle, and on the other side Subtendeth an arch more then a Semicircle.* As the right line I*B*. which on the one side subtendeth the arch I F *B*. lesse then a semicircle: and on the other side subtendeth the arch *I H B*. greater then a semicircle.

11 *A Sine is either right, or versed.*

12 *A right Sine is the one halfe of the subtense, of the double arch.*

As the right ſine of the arches B C. or B G. is the right line B E. being the ½ of the ſubtenſe of the double arches of B C. or B G. that is, the ½ of the right line B E D. which ſubtendeth the arches B C D. or B G D. So the right ſine of the arches B F. or B H. is the right line B K. that is the ½ of the right line B K I. which ſubtendeth the double arches of B F. or B H. to wit, the arches B F I. or B H I.

Conſectaries.

1 *Therefore the right ſine of an arch, leſſe or more then a Quadrant, and leſſe then a ſemicircle, is one and the ſame.* As the ſine of the arches B C. and B G. is the ſame right line B E. for that it is the ½ of the right line B E D. which ſubtendeth aſwell the arch B G D. as the arch B C D.

2 *And therupon whenſoeuer the right ſine is called the ſine of the Complement: It is vnderſtood only the ſine of the Complement of an arch leſſe then a quadrant.* As the right ſine of the Complement of B C. that is, of the arch B F. is the right line B K.

3 *Euery right ſine, is perpendiculer to the* diameter *drawne from the one tearme of the arch giuen.* For becauſe in the triangle A B D. conſiſting of 2. equall ſides A B. and A D. the ſemidiameter A G. drawne from the concurſſe of the equall ſides, cutteth the baſe B D. at E. by the definition of a right ſine, therefore that is perpendiculer to this, and this to that, by the 23. of the firſt.

4 *The right ſine of the Complement, is equall to the ſegment of the* Diameter *or* Radius *intercepted betweene the right ſine of the Arch, and the Center.* As the right ſine of the Complement B F. to wit, the right line B K. is equall to the right line E A. by the 39. of the firſt.

13 *The verſed ſine, is the ſegment of the* Diameter *intercepted betwixt the right ſine, and the Circumference.* As the verſed ſine of the Arch B C. is the ſegment of the *Diameter* E C. the verſed ſine of the arch B G. is the ſegment of the *diameter* G E.

14 *Therefore of verſed ſines, ſome are greater, and ſome leſſe.*

15 *A greater verſed ſine, is the verſed ſine of an arch, greater then a Quadrant.* As G E. is the verſed ſine of the arch G F B. being greater then a quadrant.

16 *A leſſer verſed ſine, is the verſed ſine of an arch leſſe then a Quadrant.*

Quadrant. As A C. is the verſed ſine of the arch *B C.* being leſſe then a quadrant.

17 *A* Tangent *is a right line drawne from the* Secant *by one end of the arch, perpendiculer on the extremitie of the* diameter, *paſſing by the other end of the arch.* As L C. is the *Tangent* of the arch *B C.*

18 A Secant *is a right line drawne by the one end of the arch, to the toppe of the* Tangent. As the *Secant* of the arch *B C.* is the right line A L.

19 *The definition of the quantitie which right lines have, applied to Circles, is the making of the tables of* Sines, Tangents *and* Secants. that is to ſay, of right ſines, and not of verſed: For the verſed ſines are found by the right ſines without any labour. *For the leſſer verſed ſine, with the right ſine of the Complement, is equall to the Radius.* As the leſſer verſed ſine E C. with the right ſine of the Complement A E. is equall to the *Radius A C.* Therefore if you ſubſtract the right ſine of the Complement A E. from the *Radius A C*, there reſteth the verſed ſine E C. *But the greater verſed ſine is equall to the* Radius *added to the right ſine, the exceſſe of the arch, more then a Quadrant:* As the greater verſed ſine G E. is equall to the Raduis G A. ioyned with the ſine of the exceſſe A E. Therefore if you adde the right ſine of the exceſſe *A E.* to the Radius *G A.* you ſhall haue the verſed ſine of the arch *G F B.* and therefore there is no need of the table of verſed ſines. In ſteed of the ſubtenſes, the right ſines may be vſed: for the right ſines are the $\frac{1}{2}$ of the ſubtenſes, Therefore if you take the greateſt ſine for the greateſt ſubtenſe, you may alſo take the leſſer ſine for the leſſer ſubtenſe: For the ſame reaſon is, of the halfe to the halfe, as is of the whole to the whole: As what proportion 10. hath to 6. the ſame proportion 5. hath to 3.

20 *The tables of* Sines, Tangents *and* Secants, *are commonly called the Canon of Triangles.* Rhæticus *calleth it the Canon of the doctrine of Triangles.* Vieta *the Mathematicall Canon.*

21 *The tables of* Sines, Tangents *and* Secants, *are extended no farther then to a quadrant.* For the right ſines of arches more or leſſe then a quadrant, are the ſame by the 12. *And there can be no* Tangents *and* Secants *of arches greater then a quadrant* by the 17. and 18.

22 *The tables of* Sines, Tangents *and* Secants, *are commonly*

made to minutes: Rhæticus *made them to tenths of seconds*: In the beginning and end of a quadrant, haue calculated them to minuts, seconds, or tenths of seconds, as neceſſitie required: In the reſt I haue bene contented with the ſetting downe the minutes.

23 *First of all, for the making of the tables of* Sines, Tangents, & Secants: *the Radius is to be taken of a certaine number of parts.*

24 *Of what parts ſoeuer the Radius be taken, the* Sines, Tangents, *and* Secants, *for the moſt part are all of them irrationall to it, that is, inexplicable in any true whole numbers, or fractions preciſely* by the 51. of the firſt: *And therefore the tables of* Sines, Tangents *and* Secants, *cannot be exactly made by any Meanes: yet ſuch may and ought to be made, wherein no number is different from the truth, by an integer of thoſe partes, whereof the Radius is taken.* As if the Radius be taken of 10000000. No Number of thoſe tables ought to be different from the truth by 1. of 10000000.

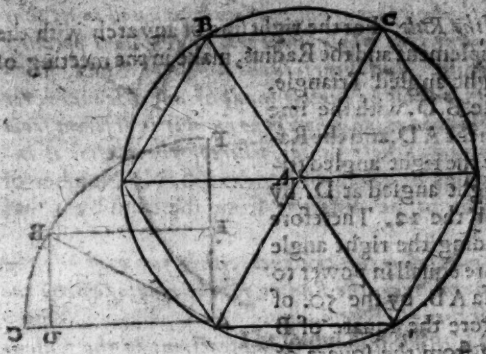
25 *That you may follow this exactines, either you muſt vſe the fractions, or elſe you muſt take the Radius, for the making of the tables, much greater then the true Radius.*

26 *But to worke with whole numbers and fractions in the Calculation is very tedious: Beſides here no fractions almoſt are exquiſitely true: Therefore the Radius for the making of theſe tables is to be taken ſo much the more, as there may be no error in ſo many of the figures towards the left hand, as you will haue placed in the tables: And as for the numbers ſuperfluous, they are to be cut off from the right hand towards the left, after the ending of the ſupputation. So did* Regiomontanus, when he would calculate the tables of ſines to the Radius of 6000000. he took the Radius of 6000000000 And after the ſupputation ended, he cut off from euery ſine found, from the right hand toward the left 4. figures: So *Rhæticus* when he would calculate a table of ſines to the Radius of 10000000000. he tooke for the Radius 1000000000000000. And after the ſupputation ended, he cutoff from euery ſine found from the right hand towards the left 5. figures: But I to finde out the Numbers in the beginning of the table, tooke the Radius of 10000000000000000000000000. But in the *Canon* it ſelfe haue taken the Radius of diuers Numbers for neceſſitie ſake: As hereafter in his place ſhall be declared.

27 *In*

27 *In the beginning you shall finde out the right Sines of all the arches lesse then a quadrant, in the same partes as the Radius was taken of what soeuer bignes: Thereupon of those right sines you shall finde the* Tangents *and* Secants.

28 *The right sines (in the making of the tables) are either primary or secundarie. The primarie Sines are those by which the rest are found.*



29 *Now I make the totall Sine, or the Radius the first primarie sine, which is equall to the side of a Sixangled figure inscribed in a Circle, that is to the subtense of 60. degrees.* Which is thus demonstrated. Let *B C.* be the side of a sixangled figure, inscribed in a Circle: Then because the arch *B C.* is *60.* partes by the Pro: therefore also the angle B A C. is *60.* parts by the *6.* of the first: And thereupon the angles A *B* C. and A C *B.* together are a 120. parts by the 49. of the first: but the angles A B C. and A C *B.* are equall by the 5. of the first: for the sides A *B.* and *A* C. opposite vnto them, are equall, that is, two Radij: Therefore either of the angles is *60.* parts: but the angle *B A C.* was also *60.* parts, Therefore the triangle *A BC.* is equiangled by the 5. of the first: but the sides *A B.* and *A C.* are Radij by the worke: and therefore the side *B C.* is Radius also: Therefore the totall sine or the Radius, is equall to the side of a sixangled figure, inscribed in a Circle, which was to be demonstrated.

30 *One*

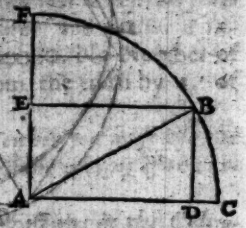
30 *Out of the totall sine I deduce all the other sines, by the 9. Problemes following.*

The first Probleme.

The right sine of an arch lesse then a Quadrant being given to finde the sine of the Complement.

The Rule: *Subtract the square of the sine given, from the square of the Radius: the square root of the remainder, is the sine of the Complement.*

The reason of the Rule. For the right sine of any arch with the sine of the Complement and the Radius, make in the meeting of the 2. sines a right angled Triangle, as the right sine B D. with the sine of the complement A D. and the Radius A B. make the right angled triangle B D A. right angled at D. by the 3. Con: of the 12. Therefore the sides, including the right angle B D. and D A. are equall in power to the hypothenusa A B. by the 50. of the first. Therefore the square of B D. being taken from the square of A B. the Remainder is the square of A D. whose square Root, is A D. or E B. the sine of the Complement, that is, of the arch F B.

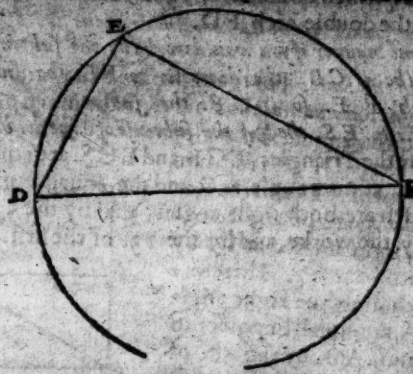


Example: Let the Radius A B. be 10000000. the sine B D. that is the sine of the arch of 30. deg. 5000000. The square of the Radius A B. is 100000000000000. The square of the sine B D. is 25000000000000. the which if you substract from the square 100000000000000, the rest shall be the square 75000000000000. whose partes shall be 8660254. the sine A D. or E B. serving for the arch F B. 60. deg.

After the same manner. The subtense of an arch lesse then a Semicircle being giuen, you may finde the subtense of the Complement to the semicircle.

For as the sine of any arch, with the sine of the complement and the Radius doe make a right angled triangle, by the 3. Con. of the 12. So the subtense of any arch with the subtense of the Complement to a semicircle and the Diameter, make a right angled

angled triangle, by the firſt Con: of the 53. of the firſt: therefore if you take the ſquare of the ſubtenſe giuen, from the ſquare of the Diameter, the Remainder ſhall be the ſquare of the ſubtenſe of the Complement: As in the Diagram propounded. If you take the ſquare of the ſubtenſe D E. from the ſquare of the diameter D F. the Remainder ſhall be the ſquare of the ſubtenſe E F.

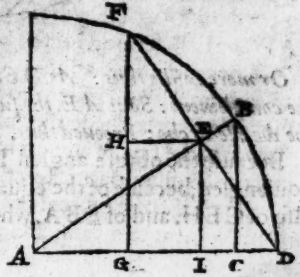


The ſecond Problem.

31 *The right ſine of an arch, being giuen with the ſine of the complement, to finde the ſine of the double arche.*

The Rule. Multiplie the right ſine of the arche, by the ſine of the complement, the product diuide by the Radius, and you ſhall haue the $\frac{1}{2}$ *of the ſine of the double arche.*

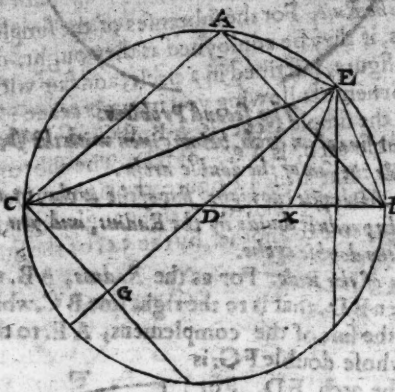
The reaſon of the Rule. For as the *Radius*, A B. to the ſine of the arche giuen B D. that is to the right line B C. which is equall to D E. So is the ſine of the complement, A E. to the right line E I. or H G. whoſe double F G. is the ſine of the arch, F D. For in the triangle, F G D. the right line H E. being parallel to the baſe G D. cutteth the ſides F G. and F D. proportionally by the 45. of the firſt. But it cutteth the ſide F D. into 2. equall parts, in E. And therefore it cutteth the ſide F G. in H. into 2. equall parts, alſo.



Exampl

Example. Let the fine of the arch B D. 35. deg. be giuen, the right line, B D. or B C. 5735764. together with A C. or A E. the fine of the complement 8191520. And let the fine of the double arche, to wit, the fine F G. be demaunded, I say: As A B. 10000000. to B C. 5735764. So is A E. 8191520. to E I. or H G. 4698462. which doubled, is 9396924. F G. the fine of 70. deg. being the double arch, F D.

After the same maner if you would worke by the subtenses: The proportion shall be as C B. the diameter to B E. the subtense of the simple arch B E. so is C E. the subtense of the Complement C A E. to E S. the ½ of the subtense of the double Arche. E B K. Because the Triangles, *E* C B. and E C S. are equiangled, because of their cómon angle at *C.* and the equall angles CEB. and C S E. which are both right angles, that by the 53. of the first, and this by the worke, and by the 23. of the first.



Or more easilie thus: As D E. the Radius to E C. the subtense of the complement: So is A E. the subtense giuen to A B. the subtense of the double arche: Prooued thus.

Because the obtuse angled Triangles, C D E. and E A B. are equiangled, because of the equall angles, E C B. and E A B. and also of C E H. and of E B A. which are equall, because the mea-

sures

ſures of them EB. AE. and CH. are equall by the Pro.

Example of this laſt manner. Let EB. the ſubtenſe of 50. de. giuen, be 8452366 together with the ſubtenſe of the complement. CE. 18126156. And let AB. the ſubtenſe of the double arche, be ſought for. I ſay.

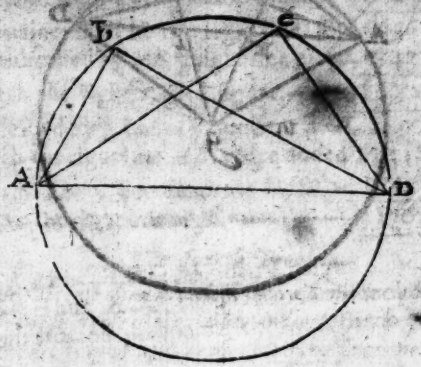
As D E. 10000000. to EC. 18126156. So is E B. 8452366. to A B. 15320890.

The third Problem.

32 *The ſubtenſe of an arch leſſe then a Semicirele, being giuen, together with the ſubtenſe of the double arche: to finde the ſubtenſe of the triple arche.*

The Rule. Take the ſquare of the ſubtenſe of the ſimple arche from the ſquare of the ſubtenſe of the double arche; diuide the Remainder by the ſubtenſe of the ſimple arch; the quotient ſhall be the ſubtenſe of the triple arch.

The reaſon of the Rule. For the ſubtenſes of the ſimple, double and triple arches, if they be conioyned as they ought, doe make a quadrilaterall figure, inſcribed in a circle, and cut with diagonals, As in the Scheme following you may perceaue: Wherein the ſubtenſe of the ſimple arche, is A B. B C. or C D. the ſubtenſe of the double arch, is A C. or B D. The ſubtenſe of the triple arche is A D. But in ſuch a figure, the right angled figure, made of the diagonals, is equall to the right angled figures made of the ſides oppoſite one to another, by the 54. of the firſt.



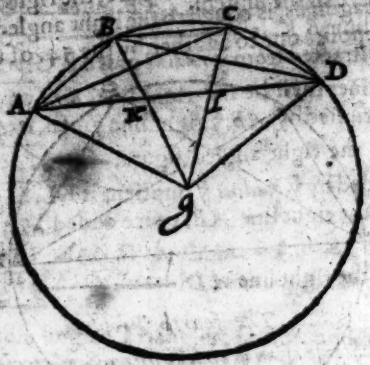
Therefore

Therefore if I subſtract the right angled figure, made of the ſides A B, and C D, that is, the ſquare of the ſimple arche, from the right angled figure made of the diagonals, that is, from the ſquare of the double arch, A C, the ſubtenſe of the double arch, there ſhall reſt the right angled figure, made of the ſides B C, and A D, which diuided by the ſide, B C, the quotient will be the ſide A D, by the 40. of the firſt: which was to be demonſtrated.

Example: let A B, or B C, the ſubtenſe of 10. deg. be 1743115. together with the ſubtenſe of 20. deg. A C, 3472964. be giuen, and let the ſubtenſe of 30. deg. A D, be ſought for.

The ſquare of the ſubtenſe A C, is	12061478945296.
The ſquare of the ſubtenſe A B, is	3038449903225.
Which ſubſtracted, reſteth the right angled, figure made of *B C*, and *A D*.	9023029042071.
which diuided by the ſubtenſe *B C*.	1743115.
the quotient is the ſubtenſe *A D*.	5176381.

Or more eaſilie without the ſubtenſe of the double arche giuen. Subſtracte the ſquare of the ſubtenſe giuen, diuided by the Radius, *from the* Radius; *the reſt multiplied by the ſubtenſe giuen, and diuided by the* Radius; *adde to the ſubtenſe 2. times taken: And you ſhall haue the ſubtenſe of the triple arche.*



The reason of the Rule, appeareth by the Scheme adioyned, wherein first the Triangles, *AGB.* and *BAK.* are equiangled, because of their common angle, *ABK.* or *ABG.* and their equall angles *AGB.* and *BAK.* which are equall by the 51. of the first: for that the arch *BCD.* which lyeth against the angle, *BAK.* or *BAD.* being in the circumference, is double to the arch *AB.*

Therefore as *AG.* to *AB.* so is *AB.* to *BK.* which subtracted, from *BG.* resteth *KG.* Then the triangles, *GBC.* and *GKL.* are equiangled, because the bases *BC.* and *KL.* are parallels by the 28. of the first. Therefore as *BG.* to *BC.* so is *GK.* to *KL.* And lastly, the Triangle *BAK.* is equiangled at the base, for it is like to the Triangle, AGB. which is equiangled at the base, as before was demonstrated: Then because the triangle *BAK* is equiangled at the base, therefore the 2. sides are equall, by the 26. of the first, and consequentlie the 2. sides *AB.* and *AK.* are equall. But the segments *AK.* and *LD.* are also equall, by the worke. Therefore if I adde *AK.* and LD. to KL. It is all one as if I should adde the right line, *AB.* twice to the right line *KL.*

Example let the same subtense, *AB.* be giuen as before, to wit, the subtense of 10 deg. 1743115. And let *AD.* the subtense of the triple arch, be sought for.

The square of the subtense *AB.* giuen, is	3038441 9903225
The right line, *BK.* is	303845
Which subtracted from ỹ *radius* giuen BG.	10000000
The Remainer shall be the right line, *KG.*	9696155
which multiplied by *AB.* ỹ right line giuen,	1743115
Produceth the right angled figure,	1690151 3222825
which diuided by ỹ *Radius* ỹ quotient is KL.	1690151
To which the right line *AB.* twice added,	1743115
	1743115
Maketh the right line *AD.*	5176381

The fourth Problem.

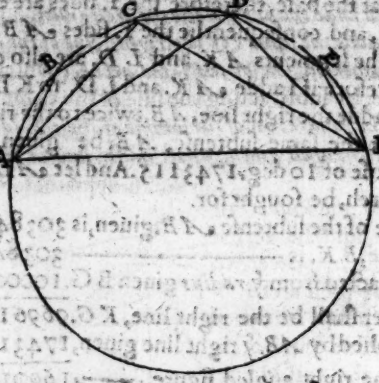
33 *The subtense of an arch lesse then a Semicircle being giuen, together with the subtense of the double and triple arche; to finde the subtence of the arche quintuple, or of an Arche fiue times as much.*

The Rule. Take the square of the subtense of the double arche, from the square of the subtense of the triple arche, the remainder diuided by the subtence giuen, shalbe the subtence of the quintuple arche.

The reason, Is the same which was in the first solution of the third Problem; For asmuch as the subtenses of the simple, double, triple, and quintuple arches, truly conioyned one with another, doe make a quadrilaterall figure, intersected with 2. diagonals, And so is to be applied to the 54. of the first. &c.

Example, Let C D. be giuen, the subtense of 2. degrees 349048. and let A F. the subtense of 10. degrees be sought for.

First the subtense of the double arch is to be found: that is, the subtense of the arch, A C. 4. deg. by the second problem. And the subtense of the triple arch, that is, the subtense of the arch, A D. by the third problem.



The subtense A C. shall be 697990. almost.
The subtense A D. shall be 1046719.
Then square those subtenses and they shalbe as soloweth.

The square of ỹ subtense of ỹ triple arch A D.	1095620664961
The square of ỹ subtense of ỹ double arch A C.	487196040100
Which substracted from the square A D. there remaine the right angled figure, made of A D. and C D.	608430624861
which diuided by the side C D.	349048
The quotient is A F.	1743114

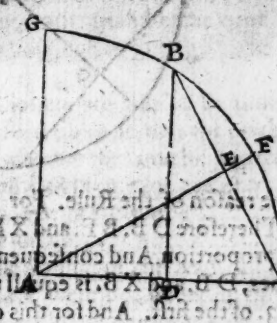
By

Note. By the ſame reaſon if neede be, you may find the ſubtenſes of the arches, 7. times, 9. times, 11. times &c. as much, as the ſubtenſe of the arch giuen. For the ſquare of the ſubtenſe of the triple arch, ſubſtracted from the ſquare of the ſubtenſe of the quadruple arch, leaueth a Number which diuided by the ſubtenſe of the ſimple arch, giueth in the quotient the ſubtenſe of an arch 7. times as much as the ſimple arch. So the ſquare of the ſubtenſe of the quadruple arch, ſubſtracted from the ſquare of the ſubtenſe of the quintuple arch, leaueth a Number, which diuided by the ſubtenſe of the ſimple arch, bringeth out in the quotient, the ſubtenſe of an arch, nine times as much as the ſimple arch. And ſo forward infinitlie.

The fift Problem.

34 *The ſine of an arch, being giuen, together with the ſine of the Complement; to finde the ſine of halfe the arch giuen.*

The Rule: *Adde the ſquare of the right ſine of the arch, giuen to the ſquare of the verſed ſine of the ſame arche,* (which verſed ſine you ſhall finde, by ſubſtracting the ſine of the complement, from the *Radius*) *The ſquare root of the ſomme of theſe 2. ſquares, ſhall be the ſubtenſe of the arch giuen, whoſe halfe ſhall be the ſine of halfe that arche.*



The reaſon of the Rule. For the right ſine, and the verſed ſine, are equall in power, to the ſubtenſe of their arche.

As in the ſcheme adioyned B D. the right ſine of the arch, B C. and D C. the verſed ſine, of the ſame arch, are equall in power, to the ſubtenſe of that arch, B C. by the 50. of the firſt, the ½ of which ſubtenſe, being E C. is the ſine of ½ the arch, being F C.

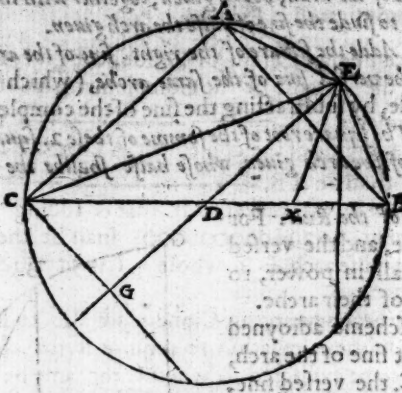
Example. Let the ſine of the arch, B C. 50. deg. be the right ſine B D. 5000000. and the verſed ſine of that arch, be D C. 1339746. The q. of the right ſine, B D. ſhall be 25000000000000. The q. of the verſed ſine, D C. ſhall be 1794919344516.

The totall of these 2. squares, shalbe 26794919344516. whose q. l. 5176380. shalbe the subtense of the arch giuen, BC. 30. deg. The ½ of which subtense, that is to say, the right line EC. 2588190. shall be the sine of ½ that arch, being FC. 15. deg.

Otherwise by the subtenses.

The Rule. Take the subtense of the complement, from the diameter, the Remainder multiplied in the Radius, *shall be the square of the subtense of halfe the arch.* As for example.

Take the subtense of the complement, A C. (equall to C X.) being the subtense of the complement of the arch giuen, A B. from the diameter C B. The Remainder X B. multiplied in the *Radius* D B. shall be equall to the square of the right line, E B. being the subtense of the halfe arch E *B.*



The reason of the Rule. For as D B. to E B. so is E B. to X B. Therefore D B. B E. and X B. are 3. right lines, in continuall proportion. And consequently, the oblong made of the extreames, D B. and X B. is equall to the q. of the Meane B E. by the 43. of the first. And for this cause it is, that as D B. to B E. so is B E. to X B. for that the Triangles, D E B. and B E X. are equiangled, because of their common angle, D B E. and their equall angles, E X B. and D E B. which are equall one to another, that is to say, for that they are equall, to a third, to wit, their common angle D B E. And therefore because the Tri-

angles,

angles, D E B. and X E B. are equicrurall, therefore they are equiangled at the baſe, by the 26 of the 1. The Triangle D E B. is equicrurall, becauſe either of the ſides, D E. and D B. is the *Radius*: The Triangle X E B. is equicrurall becauſe the right line X E is equall to the right line A E. and therefore alſo to the right line, E B. For the right lines, A E. and E B. are equall by the worke. And the right line E X. is equall to A E. becauſe they ſubtend the equall angles, A C E. and E C X. in the termes of their equall ſides; For the right line C X. is equall to the right line, C A. by the Pro: but the right line, C E. is common to both the Triangles, to wit, to the Triangles A C E. and E C X.

Therefore, becauſe the Triangles, D E B. and X E B. are equiangled, therefore it is, as D B. to B E. ſo is B E. to B X. which was to be demonſtrated.

Example.

Let the ſubtenſe of the arch, A B. 60. deg. be giuē, 10000000
together, with the ſubtenſe of the Complement, A C. 17320508
From the diameter C B. —— 20000000
I ſubſtract the ſubtenſe of the cōplement, A C or C X 17320508
the remainder ſhall be X B. —— 2679492
which multiplied by the *Radius*, D B. that is adding 7. ciphers, after this manner, 2679492 0000000 ſhall be the q. of the ſubtenſe of the halfe arch, E B. whoſe q. l. is 5176381. the ſaid ſubtenſe E B.

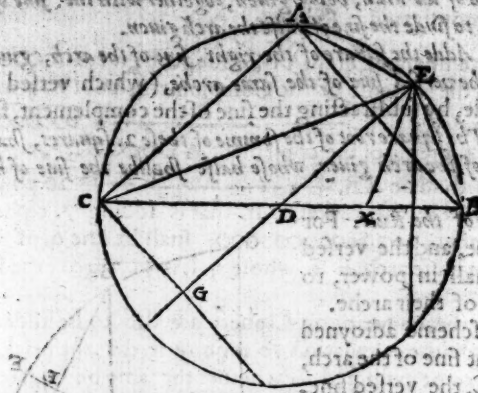
But then in theſe operations, Ciphers are alſo to be added in the beginning, if the calculation ſo require it, that the pricks of the Number to be extracted, (whether the ſame be ſquare, as heere, or cubicke, or ſolid as it will be in ſome of the examples following.) may dulie be noted. For the Numbers from the right hand, if a great *Radius* be taken, are not alwayes to be written downe. In which caſe the noting of the Radicall prickes ſhould be vncertaine, if ciphers were not added in the beginning. But this adding of Ciphers in the beginning, hath another vſe, for it ſheweth that all theſe ſubtenſes are leſſe then the *Radius*, and as it were certaine parts of the *Radius*, which parts are commonly thus written, $\frac{5176381}{10000000}$ But much more breefe, and neceſſary for the worke, is this writing of it: 5176381. For

The totall of these 2. squares, shalbe 2679491934451 6. whose q. l. 5176380 shalbe the subtense of the arch giuen, BC. 30. deg. The ½ of which subtense, that is to say, the right line EC. 2588190. shall be the sine of ½ that arch, being FC. 15. deg.

Otherwise by the subtenses.

The Rule. *Take the subtense of the complement, from the diameter, the Remainder multiplied in the* Radius, *shall be the square of the subtense of halfe the arch.* As for example.

Take the subtense of the complement, A C. (equall to C X.) being the subtense of the complement of the arch giuen, A B. from the diameter C B. The Remainder X B. multiplied in the *Radius* D B. shall be equall to the square of the right line, E B. being the subtense of the halfe arch E *B.*



The reason of the Rule. For as D B. to E B. So is E B. to X B. Therefore D B. B E. and X B. are 3. right lines, in continuall proportion. And consequently, the oblong made of the extreames, D B. and X B. is equall to the q. of the Meane B E. by the 43. of the first. And for this cause it is, that as D B. to B E. so is B E. to X B. for that the Triangles, D E B. and B E X. are equiangled, because of their common angle, D B E. and their equall angles, E X B. and D E B. which are equall one to another, that is to say, for that they are equall, to a third, to wit, their common angle D B E. And therefore because the Tri-

angles,

angles, D E B. and X E B. are equicrurall, therefore they are equiangled at the base, by the 26 of the 1. The Triangle D E B. is equicrurall, because either of the sides, D E. and D B. is the *Radius*: The Triangle X E B. is equicrurall because the right line X E is equall to the right line A E. and therefore also to the right line, E B. For the right lines, A E. and E B. are equall by the worke. And the right line E X. is equall to A E. because they subtend the equall angles, A C E. and E C X. in the termes of their equall sides: For the right line C X. is equall to the right line, C A. by the Pro: but the right line, C E. is common to both the Triangles, to wit, to the Triangles A C E. and E C X.

Therefore, because the Triangles, D E B. and X E B. are equiangled, therefore it is, as D B. to B E. so is B E. to B X. which was to be demonstrated. *Exemple.*

Let the subtense of the arch, A B. 60. deg. be giuē, 10000000
together, with the subtense of the Complement, A C. 17320508
From the diameter C B. —— 20000000
I substract the subtense of the cōplement, A C or C X 17320508
the remainder shall be X B. —— 2679492
which multiplied by the *Radius*, D B. that is adding 7. ciphers, after this manner, 2679492.0000000 shall be the q. of the subtense of the halfe arch, E B. whose q. l. is 5176381. the said subtense E B.

But then in these operations, Ciphers are also to be added in the beginning, if the calculation so require it, that the pricks of the Number to be extracted, (whether the same be square, as heere, or cubicke, or solid as it will be in some of the examples following,) may dulie be noted. For the Numbers from the right hand, if a great *Radius* be taken, are, not alwayes to be written downe. In which case the noting of the Radicall prickes should be vncertaine, if ciphers were not added in the beginning. But this adding of Ciphers in the beginning, hath another vse, for it sheweth that all these subtenses are lesse then the *Radius*, and as it were certaine parts of the *Radius*, which parts are commonly thus written, $\frac{1}{10000000}$ But much more breefe, and necessary for the worke, is this writing of it; 5176381. For

those numbers are altogether of the same value, as these 2. numbers, 09. and $\frac{9}{10}$ are.

Yet otherwise by the subtenses, and by Algeber, of the inuention of *Iustus Burgius*.

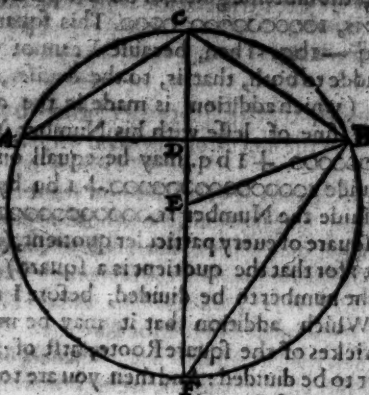
He that knoweth not *Algeber*, let him leane the Algebraicall worke here, and through out the whole booke, for these examples are not put of necessitie, but onely of curiositie.

The rule. *Diuide the square of the subtense, giuen by* 4q—1bq. *the quotient shall be the* q. *of the subtense, of halfe the arche.*

The reason of the rule. For the square of the subtense of any arch whatsoeuer, shall be equall to 4. squares, lesse by one biquadrat of the subtense of halfe the arch. Which is thus demonstrated.

Let the right line, A B. be giuen for the subtense of the arch, A C B. And let the subtense of $\frac{1}{2}$ that arch, to wit, the right line, A C. or C B. be demaunded. Let the diameter F C. be 2. that the *Radius*, may be made 1. as it is put in the tables of sines, although there, many ciphers be added to 1. which heere there is no neede of. Then let the subtense A C. or C B. being demaunded, be put algebraically for 1. roote or side: and so is C B. 1. Root: therefore the square of C B. shall be 1q. for 1. l. multiplied by 1. l. giueth 1q. If you take this square from the square of the diameter 2. to wit, from 4. there shall rest, 4—1q. which is the square of the right line F B. by the 50. of the 1. Because the Triangle, F C B. is right angled at B. by the 1. corfect. of the 31. of the 1. Therefore the right line, F B. is the roote of the square, of 4—1q. which roote may be thus noted, 4.—1q. or so: 14.—1q. as euery one hath accustomed himselfe. Let also the *Radius*, E B. be drawne to make the Triangle E F B. Now the triangles, E F B. and A C B. are equiangled because of their equall angles, C F *B*. and C A *B*. which are equall, because of their equall, or rather the same measure, which is the arch, C *B*.

But the angle C *B* A. is equall to the angle, C A *B*. and the angle E *B* F. is equall to the angle C F *B*. or E F *B*. by the worke: therefore



therefore also the third angle, ACB. is equall to the third angle, FEB. by the 4. conſect: of the 49. of the 1. Then, becauſe the triangles, EFB. and ACB. are equiangled, therefore is it, as EF. 1. to FB. l. 4—1q. ſo is AC. 1. l. 1q. to AB. In which worke, that you may multiplie the ſecond tearme, by the third, becauſe the ſecond tearme, is a ſurd number, make the third tearme alſo a ſurd number, by multiplying 1. l. by it ſelfe to make 1q. which being done, the right line AC. ſhall be l. 1q. Then multiplie l. 4—1q. by l. 1q. after this manner.

The Number to be multiplie; l. 4—1q.
The Number multiplying —— l. 1q.
The product —— l. 4q—1bq.

Therefore the ſubtenſe AC. is equall to the Roote of foure ſquares, leſſe by one biquadrat of the roote aſſumed AC. And conſequently the ſquare of the ſubtenſe AB. is equall to foure ſquares leſſe by one biquadrat of the aſſumed roote, or of the ſubtenſe of halfe the arch, AC. or CB. And ſo if you diuide the ſquare of the ſubtenſe, AB. by 4q.—1bq. the quotient ſhall be the ſquare of the ſubtenſe of the halfe arch, AC. or CB. which was to be demonſtrated.

But how the ſquare of any ſubtenſe giuen, may be diuided by 4q.—1bq. I will make manifeſt in the example following.

Example. Let the subtense giuen, of 60 de. *A B*, be 10000000 whose square is, 100000000000000. This square is to be diuided by, 4q —1bq. Then, because I cannot well diuide by 4q—1bq. I adde to both, that is, to the deuiser, and to the diuidend 1bq. (which addition is made in the diuiser, by taking away the sine of Iesse with his Number,) that 4q. and 100000000000000. + 1bq. may be equall one to another. And then I diuide 100000000000000. + 1bq. by 4q. or which is all one, I diuide the Number 100000000000000. by 4. So as I adde the square of euery particuler quotient, (which in truth is a biquadrat: for that the quotient is a square) with his complement, to the number to be diuided, before I moue forward the deuiser. Which addition that it may be made in his due places, the prickes of the square Roote, first of all is to be put to the number to be diuided: And then you are to proceed after the same manner as followeth.

The square of the subtense, of 60. d. *A B*. is 10000000000000

(4) Say 4. in 1 (0

4. in. 10. (2

2 added. 4. to 100 is

makes 104. Say twice 4. is 8. which substract

rests 2400

(4) Say 4. in 24. (6

D 276. added

2676

24 substracted

rests 27600

(4) (7

3689 added

31289

28 substra

rests 328900

(4) — (9

48141 added

The square of 4. is 4. D.

The square of 6. is 36.

The Complement is 24.

The totall is 276. D.

And that Complement is found by multiplying the Roote 6. by the double of 2. the Roote going before, that is by 4. For 4. times 6. is 24.

Hence forward the Numbers, to be substracted are

not

not put downe, euen as they are not vsed to be put down, in vulgar diuision.

rests 1704100
(4) (4
214336 ad

rests 318436 00
(4) (9
4820001 ad

rests 66660100
Ad. (4) 5358981 (1

rests 3201908100
(4) (9
Ad. 482308401

rests 8421656100
(4) (2
Ad. 1071796764

rests 149345286400
(4) (4
Ad. 2143593 5376

rests 10781221776 00
(4) (3
Ad. 1607695 15449

rests 3889169304900 (1
Ad. (4) 45358983848 61

rests 42506768976100 (1
Ad (4) 5358983848621

rests 786575282472100
(4) (2
Ad. 107179576972444

937549594445 44. 2 should follow.

The proceſſe of the perticuler biquadrants, from whence the whole biquadrate is leaſurely compoſed.

$$\begin{array}{l} 00\ \binom{2}{2} \\ \hline A.\ 4\ (6 \\ \quad 46 \\ B.\ 276 \\ \hline \quad 676\ (7 \\ \quad\quad 527 \\ C.\ 3689 \\ \hline \quad 71289\ (9 \\ \quad\quad 5349 \\ \quad\quad 048141 \\ \hline \quad 7177041\ (4 \\ \quad\quad 53584 \\ \quad\quad 214336 \\ \hline \quad 7179184 36\ (9 \\ \quad\quad 535889 \\ \quad\quad 4823001 \\ \hline \quad 71796666601\ (1 \\ \quad\quad 5358981 \\ \hline \quad 717967201981\ (9 \\ \quad\quad 535898 29 \\ \quad\quad 482308461 \\ \hline \quad 7179676842 16561\ (2 \\ \quad\quad 535898382 \\ \quad\quad 1071796764 \\ \hline \quad 7179676949345 2864\ (4 \\ \quad\quad 5358983844 \\ \quad\quad 2143593537 6 \\ \hline \quad 7179676970781221776\ (3 \\ \quad\quad 535898384 83 \\ \quad\quad 160769515449 \\ \hline \end{array}$$

A. The ſquare of the Roote. 2.

B. The ſquare of the Roote 6. with his Complement. The ſquare of the Root 6. is 36. The complement made by the multiplycation of double the roote precedent, and this Roote 6. is 24. Theſe after their due order (as is afore ſhewed) added together, make 276.

C. The ſquare of the Roote 7. with his complement made by multiplying the double of 26. the Roote aforegoing in this Roote 7.

```
717967697238891693049 (1
                       535898384861
-----------------------------------
717967697244250676897761 (1
                       535898384861
-----------------------------------
717967697244786575182471 1(2
                       535898384861222
                       107179676972444
-----------------------------------
717967697244893754959444544. the whole biquadrant.
```

The proofe of the former Resolution, by changing it contrarily.

```
1 q.  26794919243112
4 q.  107179676972448
1 bq.  7179676972448  Substracted
-----------------------------------
```

Resteth 1000000000000000. the square of the subtense of 60. degree.

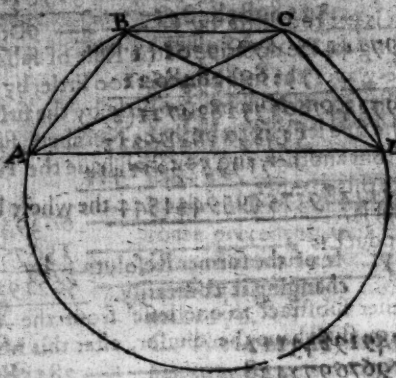
The seauenth Probleme.

The subtense of an arch being giuen, to finde the subtense of the third part of that arch.

The Rule. *Put the third part (of the subtense giuen) somewhat augmented, for the subtense demaunded, and by that subtense, finde out the subtense giuen, according to the doctrine of the third Probleme: Which if you finde the same, you haue that you sought for: But if otherwise, note the difference by more or lesse, and when you haue repeated the same worke, by an other position of the subtense sought for. Againe, note the difference by more or lesse: Which being done, afterwards by the rule of false, you shall infallibly finde the truth.*

The reason (why the third part, and somewhat more of the subtense giuen, may be put probably for the subtense sought for) is thus: because the subtense of the third part of the arch three times taken, is of greater necessitie then the subtense of the triple arch: As for example, the three right lines A B. B C. and C D. put together, are of necessitie greater then the right line A D.

The



The reaſon of the reſt of the worke vntill you come to the rule of falſe, appeareth by the demonſtration of the third Problem.

The reaſon of the rule of falſe, is knowne from Arithmeticians. *Example*, Let the ſubtenſe of 30. degrees be giuen A D. 5176381. almoſt, And let the ſubtenſe of $\frac{1}{3}$ part of that arche, that is to ſay, the ſubtenſe of the arch of 10. degrees A B. be demaunded.

The ſubtenſe giuen A D. is	5176381.
The $\frac{1}{3}$ part thereof is	1725460.
That ſomwhat augmented, is	1730000.
Or	1740000.
Or	1750000.
Let the firſt poſition be	1730000
Whereby the ſubtenſe of the triple arch A D. ſought for according to the doctrine of the third Problem, ſhall be	5138223.
But it ſhould be	5176381.
Therefore it is too little by	38158.
Let the ſecond Poſition be	1740000.
Whereby the ſubtenſe ſought for of the triple arch A D. according to the doctrine of the third Problem, ſhall be	5167320.

But

But it ſhould haue bene ——— 5176381.

Therefore it is yet too little by ——— 9061.

Now according to the doctrine of the Rule of falſe poſition, Multiply croſſe-wayes, the firſt Number too little, by the ſecond poſition, and the ſecond Number too little by the firſt poſition. And becauſe the ſine leſſe, is to both the Numbers, ſubſtract the products one from another, and you ſhall haue the Number to be deuided after this manner.

The firſt product is ——— 66394920000.

The ſecond product is ——— 15675530000.

The diuidend is ——— 50719390000.

In like manner ſubſtract th'one leſſe from the other leſſe Number, and you ſhall haue the diuiſor, after this Manner.

The one leſſe ——— 38158.

The other leſſe is ——— 9061. ſubſtracted

Reſteth the diuiſor ——— 29097.

The diuiſion is ſelfe.

The dividend 50719390000

The diuiſor 29097 (1 — 1

216223 (7 — 8

29097

203679

125449 (4 — 3

29097

116388

90610 (3 — 6 X 0

87291

33190 (1 — 7

29097

40930 (1 — 8

29097

118330 (4 — 3 X 0

29097

116388

1924

The

The diuision being ended, the quotient, as you see, is the Number 1743114. for the subtense A B. By which Number worke againe as by the first or second position, and the quotient will be againe too little, but very little, to wit, 3. Therefore take a number a little greater then the Number 1743114. to say the Number 1743115. and keeping the former worke, you shall finde the subtense A D. such as the giuen subtense was in the beginning: that is 5176381. which notwithstanding in the conclusion, is more then the truth: And therefore also, the subtense 1743115. in the conclusion will be more then the truth; yet more neere the truth then the subtense 1743114. as by the worke appeareth: because truly it leaueth no apparent difference betweene the subtense giuen A D. and the subtense sought for.

Note. The Rule of false alwaies sheweth you the truth in the least part, in twice as many more Ciphers, as the first or second position had signifying figures (as 1.2.3.4.5.6.7.8.9. but not 0.) towards the last cipher. As for example: In the aforegoing example; either position seuerally taken, had three signifying figures towards the last, to wit, 173. or 174. Therefore the Rule of false shall exactly produce the truth in sixe ciphers, to say in these 1743114. whence it appeareth, If you make the first position 1743114000000 the second 1743115000000. you shall in the end haue the true subtense exactly to the Radius 1000000000000. But if againe, you shall increase this subtense with 14. ciphers, and shall take the last signifying figure in the one position lesser, and in the other greater, you shall finde very exactly the true subtense demaunded to the Radius: 10000000000000000000000000000 and so for the rest, so that you had the subtense *A D.* first giuen in so many parts.

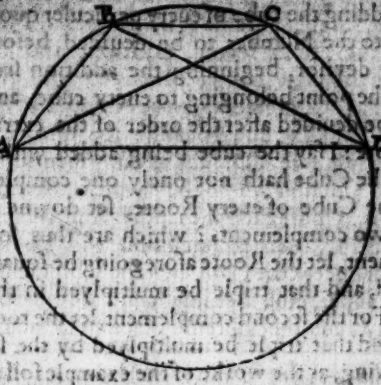
Otherwise by *Algebra*.

The Rule. *Diuide the subtense giuen by* 3. *lesse* 1 c. *the quotient shall be the subtense of the third part of the arch giuen.*

The reason of the Rule. For the subtense of whatsoeuer arch is equall to 3. Roots lesse 1. Cube, of which Rootes, one Roote is the subtense of the third part of that arch.

Which is thus demonstrated: Let the subtense giuen be *A D.* being the subtense of the arch *A B C D.* and let the subtense

A B. or



AB. or *BC.* or *CD.* being the ſubtenſe of the third part of that arch, be ſought for: Let BC the ſubtenſe of the third part be put for 1.l. Then the ſubtenſes of the double arches, to wit, the right lines AC. and BD. euery of them ſhall be to the arch BC, l. 4.q — 1.bq. by the demonſtration of the Problem afore going. But the figure ABCD. is a foure ſided figure inſcribed in a circle, and interſected with Diagonalls. Therefore the right angled figure made of AC. and BD. is equall to the right angled figures (of the ſides) added together by the 54. of the firſt: Firſt then I then multiply the Diagonalls together, and thereof is made the ſquare 4. q. — 1. bq. (For to multiply a ſurd Number by it ſelfe, is nothing els but to take away the ſigne l.) After I multiply the ſide AB. by the ſide CD. that is 1 l. by 1 l. & therof is made 1 q. which I ſubſtract out of the ſquare of the diagonalls, that is out of 4 q. — 1 bq. & the reſt is 3 q. — 1 bq. for the right angled figure made of BC, and AD. which right angled figure 3 q — 1 bq. if I diuide by the ſide BC. that is by 1 l. the quotient is the ſide AD. 3 l. — 1 c. Therefore 3 l. — 1 c, whoſe ſides being the ſubtenſe of one third part, is equall to the ſubtenſe of the arch giuen; and conſequently, if the ſubtenſe of the giuen arch be deuided by 3 l. — 1 c, the quotient ſhall be the ſubtenſe of one third part, which was to be demonſtrated.

The

The manner of this deuision is thus: Deuide the ſubtenſe giuen, by 3. adding the cube of euery perticuler quotient with his complement to the Number to be deuided, before you moue forwards the deuiſor, beginning the addition from the right hand, vnder the point belonging to euery cube, and put to the Number, to be deuided after the order of the extraction of the Cubicke Roote: I ſay the cube being added with his complements: For the Cube hath not onely one complement, as the ſquare, but the Cube of euery Roote, ſet downe after another Roote hath two complements? which are thus found: for the firſt complement, let the Roote aforegoing be ſquared, and that ſquare tripled, and that triple be multiplyed in the roote next following: For the ſecond complement, let the root aforegoing be tripled, and that triple be multiplyed by the ſquare of the Roote following, as the worke of the example following, which I haue ſet downe at large, ſheweth.

Example.

The ſubtenſe of 30. deg. giuen, from whence the ſubtenſe of 10. deg. is to be drawne.

0.517.638.1
(3)(3)1 Adde (01
218 638 (7
(3)3 913. A.
12 551 100 (4
(3) 355 024.A.
906 124 000 (3
(3)27 295 407.A.
33 419 407 000 (1
(3) 911 466 991.A.
4 330 873 991 (1
(3) 91 152 451 231.A.
1 422 026 442 231 (4
(3)36 461 273 334 544.A.
258 487 715 565 544. There ſhould follow (8)
(3)

The

The perticuler squares of the Subtense of 10. degrees.

0.1
C.0001.1
91
3913
0001.7
27
189
0000491.3(7
355024
0000289.4
144
1378
0000521.8024(2
272954.7
0030276.3
3483
10449
0005295319407(3
911466991
0030384049.1
34861
0005296230873991(1
911524512.31
0030383976.1.1
348621
0005296322026442231(1
3649127333454.4
0030384324721.4
3486224
13944896
0005296358487715565544(4

The finding out of the perticuler Cubes of the subtense of 10. degrees.

0030384464169996

1.1	1.17	1.4
q.1	q.289	q.16
3	3	51 C.64
3	867	q.16
1.7	1.4	306
1 Comple. 21	1. Compl. 3468	51
2 Comple. 147	2. Compl. 816	816
the Cube 343	The cube. 64	

3913 the cube of the roote 7. with his complements.

355024. the Cube of the roote 4. with his complements.

l. 1743 l. 1743 l. 1743 l. 1743

q. 30276 3 q. 9 q. 30380 3 3

522 c. 27 3 5229

90828 q. 9 9114147

5229

l. 3 4698 1

272484 911466991

4698

27

272954071

l. 174311 l. 174311 l. 174311 l. 4

3 3 q. 16

q. 303839761 q. 30384324721

3 52293 3 522933 C. 64

q. 16

911519283 911529741631

52293 4 3137598

1 522933

364611896652

911524531231 8366928 8366928

64

364612733344544

The proofe of the aforegoing worke, by composition contrarily.

11. 0174311|8 The Subtenſe of 10 degrees.

3l. 0522934|4

1C. 0005296|3|5 Subſtract.

0517638|0|9 Remaineth for the ſubtenſe of 30. deg.

The

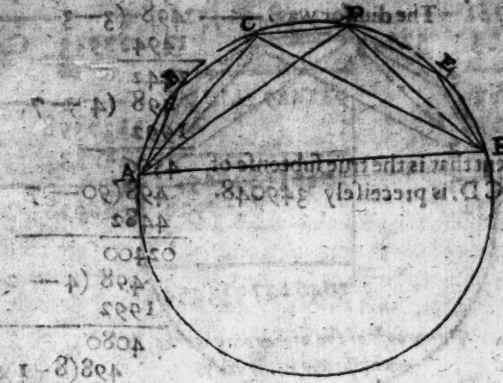
The ſeauenth Problem.

36 *The ſubtenſe of any arch being giuen, to finde the ſubtenſe of the fift part of that arch.*

The Rule. *Put for the ſubtenſe demaunded ſome what more then the $\frac{1}{5}$ part of the ſubtenſe giuen, and thereby finde out the ſubtenſe giuen, according to the doctrine of the fourth Problem: whereby if you finde the ſame, & such as you deſire: But if otherwiſe, note the difference by more or leſſe, and the ſame marke withall by an other poſition of the ſubtenſe demaunded. Againe, more or leſſe. And laſtly, finde out the truth by the rule of falſe poſition, as in the 6. Problem.*

The reaſon of the rule is the ſame which was in the ſixt Problem.

Example. Let A F. 1743115. the ſubtenſe of 10. degrees be giuen: And let the ſubtenſe of the fift part, that is the ſubtenſe of 2 deg. C D. be demaunded.



The ſubtenſe of 10 deg. is — — 1743115. almoſt
The $\frac{1}{5}$ thereof is — — — 348623.

Let the firſt poſition be — — — 349000.
Thereby A F. is found — — — 1742875
But it ſhould be — — — — 1743115.

Therefore

Therefore it is lesse — — 240.

Let the second Position be — — 249100.
Thereby A F. shall be — — — 1743373.
But it ought to be — — — 1743115.

Therefore it is more + 258.
To which adde the lesse — 240.

And you shall haue for Diuisor — 498.

Then multiplie crosse-wayes, that is the lesse by the greater position: And the more, by the lesser position. And then
The first product shall be — — 83784000.
The second product shall be — 90042000.

These adde & you shall haue the diuidend. 173826000

The diuisor was — 498 (3—3
1494
2442
498 (4 — 7
1992
4506

The Quotient that is the true subtense of 2. degrees C D. is preceisely 349048.

498(90 — 7
4482
02400
498 (4 — 2
1992
4080
498(8 — 1 × 3
3984 0
96

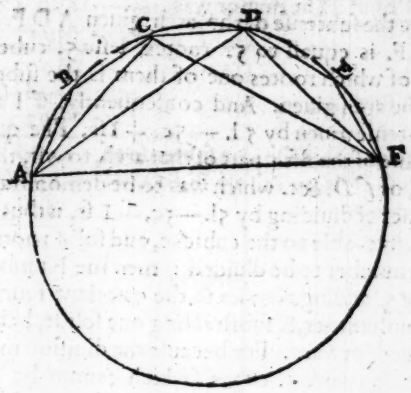
Otherwise by Algebra.

The rule. *Diuide the subtense giuen by 5 l. — 5 c. + 1 q. the quotient shall be the fift part of the arch giuen.*

The

The reaſon of the rule. For the ſubtenſe of any arch whatſoeuer, is equall to fiue rootes leſſe, fiue cubes more one ſolide; one of whoſe rootes is the ſubtenſe of the fifth part of that Arch: Which is thus demonſtrated.

Let A F. the ſubtenſe of the arch A B F. be giuen, and let the ſubtenſe of the fifth part of the arch A B F. be demaunded, that is the ſubtenſe of the arch C D. to wit, the right line C D. Let C D. be put for one Roote: or which is all one, Let C D. be 1 l. Therefore A C. ſhall be 1 4 q. — 1 bq. And likewiſe D F. by the demonſtration, of the 31. Problem. But A D. ſhall be 3 l. — 1 C. And ſo alſo C F. by the demonſtration of the Problem next afore-going. Now in the quadrilaterall figure A C D F. interſected by the diagonalls A D. and C F. the right angled figure of the diagonalls A D. and C F. is equall to the right angled figures (made of the oppoſite ſides, to wit, of C D. and A F. and alſo of A C. and D F.) added together by the 54. of the firſt.



Firſt then I multiply the diagonalls A D. and C F. together: Then I multiply the ſides oppoſite A D. and D F. together: And I ſubſtract the product of this multiplycation, from the product of the diagonalls. The remainder is the right angled figure, of th'other two oppoſite ſides C D. and A F. which right angled

figure diuided by the right line C D. leauing the right line A F.

The whose Algebraicall worke is thus.

A D. 3 1 — 1 C.	A C. 1 4 q — 1 bq.
C F. 3 1 — 1 C.	D F. 1 4 q — 1 bq.
9 q — 3 bq.	A C. D F. 4 q — 1 bq.
— 3 bq + 1 q C.	
A D. C F. 9 q — 6. bq + 1 q C.	
A C. D F. 4 q — 1 bq.	
C D. A F. 5 q — 5 bq + 1 q C.	
The Deuisor C D. 1 1.	
The Quotient A F. 5 l. — 5 C + 1 s.	

Therefore the subtense of the arch giuen A D F. to wit, the right line A F. is equall to 5. rootes, lesse 5. cubes, more by one solide: of which rootes one of them is the subtense of the fift part of the arch giuen. And consequently, if I shall diuide A F. the subtense giuen by 5 l. — 5 c. + 1 s. The quotient shall be the subtense of the fifth part of that arch, to wit, the right line A B. or B C, or C D. &c. which was to be demonstrated.

The manner of diuiding by 5 l. — 5 c, + 1 s. is thus: First, of all the points agree-able to the cubicke, and solid roots, are to be put ouer the number to be diuided: then the Number diuidend is diuided by 5. adding alwaies to the quotient found 5. cubes with his complements, & substracting one solide, before the deuisor be moued forward. For because the diuision must be made by 5 l. — 5 c. therefore 5. cubes (which cannot be taken from the deuisor) are contrarily to be added to the number to be diuided. In like manner, because the diuision must be made by 5 + 1 s. Therefore one solide (which cannot be added to the diuisor) is to be substracted from the number to be diuided.

Example. *Let the subtense of 10. deg. be given, one of which is to be extracted the subtense of 2. degrees.*

S.
C. 2c. S. C.
0174 31 1 4
(5) (5) 13 5. Adde
44 6 400 0
21 3
24 44 6 375 7
(5) 6 1 520 Adde
4 507 7 895 7
21 1
4 507 7 874 5
(5) 16 022 7
23 897 3
6 3
23 890 9
(5) (5) 73 20
3 964 0
3. 964 0
(5)

(3
Substr.
(4
35 424 Substr.
64 576
45 Adde (9
09 576 000 00
40 413 767 49 Substr.
69 162 232 51
88 976 320 Adde (4
58 138 552 51 0000000000
29 677 769 34 9411073024 Sub.
28 460 783 16 5389269760
(8 almost

The perticuler squares of the subtense of 2. degrees.

0.0.(3
00003
00009
00009(4
64
256
00011 56.(9
689
6201(0

00012180

000121801100 (4
69804
279216
000121828921 6 (8
698088
5584704
1218345003 04

The perticuler subtenses of the cube of two degrees.

C. 0000027 Radix (3
12 304
00000 39 304 (4
3 204 349
0000042 598 549 (90
14 617 795 264
0000042 523 166 795 264 (4
2 923 901 134 592
0000042 526 090 756 398 592. one Cube (8
5. the Multiplyer.
0000212 630 453 781 992 960. 5. Cubes.

The perticuler Solides of the subtense of 2. degrees.

0000000243 —— the Roote (3
211 35424
454 35424 (4
63 40413 76749
517 75837 76749 (90
29677 76934 94110 73024
518 05515 53683 94110 73024 (4
5937 18660 97927 80583 &c.
518 11452 72344 92038 53607 (8

The

The proofe of the Resolution made by contrary *Compositian.*

1. l. 00348048. the subtense of 2. deg.

5. l. 01745240.

— 5. c. 00002126. subtract.

01743114.

+ 1 l. 000000005 l. &c. Adde if any thing be to be added

1743114. the totall is the subtense of 10. deg.

But how the perticuler Cubes are to be found, was shewed in the example, of the aforegoing problem. Nor is this any new thing, if that euery perticuler cube, be not multiplied by 5. before they be added, because here 5. cubes are to be added to the Number to be diuided: as for example, the first perticuler cube in this example was, 27. this multiplied by 5. maketh 135. The other perticuler cube, with his complements, was 12304. This number multiplied by 5. maketh 61520. And so forwards.

The perticuler solids, you shall finde thus: A solide is made by the multiplication of a Cube, by a square: As the solide of 3. is thus made: 3. times 3. is nine, and thrice, 9. is 27. and 9. times, 27. is 243. And this is the making of the solide of one figure, or else of more figures, considered ioyntlie together: As the solide of 34. is 45435424. For the square of 34. is 1156. the cube is, 39304. which two multiplied together, make the Number 45435424. But if you would finde, out the solides of euery figure with their complements seuerally, that is to say: If after the finding of the solide, of 3. you would finde the solide of 4. which, with his complements added to the solide of 3. maketh the solide of 34. you must worke an other way, as thus: The solide of more figures, as for example: the solide of 34. to all the figures after the first, hath foure complements: as there are 4. figures put betweene euery of the points of the solide rootes: those 4. complements you shall thus finde; For the first complement, you shall first square the square of the roote afore-

going, then you shall multiplie that biquadrat by 5. And that product you shall multiplie by the present roote. For the second complement you shall cube the roote aforegoing, and multiplie that number, by 10. and that last product you shall multiplie by the square of the present roote. For the third complement, you shall square the roote aforegoing, and multiplie that square by 10; And that product you shall multiplie by the cube, of the present roote. For the fourth complement, you shall multiplie the roote aforegoing, by 5; And that product you shall multiplie by the bi-quadrat of the present roote. Then multiplie the present roote into a surfolide; And lastlie adde those 5. Numbers together, vnder writing one vnder an other in such manner as the example following sheweth.

The solide of 4. in the roote 34.

	l. 3.	l. 3.	l. 3.	l. 3.	l. 4.
	q. 9.	q. 9.	q. 9.	5.	10.
	bq. 81.	c. 27.	90.	15.	c. 64.
	405.	270.	5. 64. bq 256.		10.
	l. 4.	16.	360.	90.	3841.
1. *Complement.*	1620.	1620.	540.	75.	64.
2. *Complements.*	4320.	270.	5760.	30.	5. 1024.
3. *Complement.*	5760.	4320.		3840.	
4. *Complements.*	3840.				
5. *of 4. is.*	1024.				

21135424. The perticuler solide, of 4. with his complements in respect of 3. the roote aforegoing.

Note: By the same manner of worke, you may finde the subtense of the seuenth, ninth, eleuenth, thirteenth, and infinitly of any part whatsoeuer of any vneuen Number, if need require.

The eight Problem.

37 *The sines of 2. vnequall arches, being giuen together with the sines of their complements, to finde the sine of the somme, or of the difference of those arches.*

The

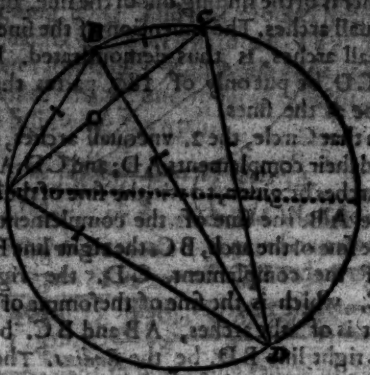
The rule. *Multiplie alternately the fine of the one arche by the fine of the complement of the other: If you adde the products together and diuide that totall by the* Radius, *by cutting off figures from the right hand, you shall haue the fine of the somme of the 2. giuen arches: But if you substract the lesse product from the greater, you shall haue the fine of the difference of the same arches.*

The reason of the Rule. This rule hath 2. Members, one is of the finding out of the fines of the somme, of two vnequall arches: The other is of the finding out of the fine, of the difference of two vnequall arches. The inuention of the fine of the somme of 2. vnequall arches, is thus demonstrated. Let the whole Circle, A B C D. be put only of 180. partes that all the subtenses may be as the fines;

And let in that Circle, the 2. vnequall arches, giuen be A B; and B C. and their complements A D; and C D. And let the fines of all these arches be giuen, to wit, the fine of the arch A B; let be the right line A B. the fine of the complement A D; the right line A D. the fine of the arch, B C. the right line B C. the fine of the fine of the complement, C D. the right line C D. And let A C. which is the fine of the somme of the 2. giuen arches, that is of the arches, A B and B C. be sought for. But let the right line B D. be the *Radius*. Then because the said fines in this manner inscribed in a Circle make a quadrilaterall figure, intersected with diagonals, in which figure the right angled figure made of the diagonals, is equall to the 2. right angled figures, added together made of the opposite 2. sides by the 54. of the first. Therefore if you multiplie the fine A B. by the fine of the complement of the arch B C. that is by the opposite side C D; And likewise the fine of the arch, B C; by the fine of the complement of the arch, A B; that is by the opposite side, A D; and adde those 2. products together, you shall haue a right angled figure, equall to the right angled figure of the diagonals A G; and B D. which right angled figure if you diuide by the knowne side, to wit, by the *Radius*, B D. the quotient shall produce the vnknowne side A C. to be the fine of the somme of the arch, *A B.* and *B C.* or the fine of the arch *A C.* which was to be demonstrated.

The inuention of the fine of the difference of 2. vnequall arches,

arches, is thus demonstrated. Let the sine of the greater arch, *AB*, be the right line *AB*, and the sine of the lesser arch, *BC*, be the right line *BC*. Let the sine of the difference, *AC*, be the right line *AC*, the sine of the complement of the greater arch, *AB*, let be the right line *AD*. And the sine of the complement of the lesser arch, *BC*, be the right line *CD*, being the Radius. Then againe because the figure *ABCD*, is a quadrilateral figure inscribed in the Circle, and intersected with diagonales

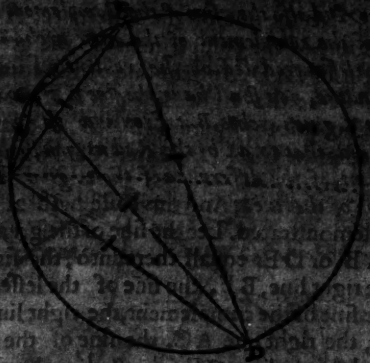


Therefore if I multiplie the sine of the greater arch, *AB*, that is the right line, *AB*, by the sine of the complement of the lesser arch, *BC*, that is by the right line *CD*, I shall haue a right angled figure, made of the diagonalls, equall to the 2. right angled figures made of the 2. opposite sides.

Moreouer, if I multiplie the sine of the lesser arch, BC. to wit, the right line BC. by the sine of the complement of the greater arch, AB. to wit, by the right line, AD. And if I take this product of the 2. opposite sides, BC. and AD. from the right angled figure made of the diagonall, AB. and CD.

There shall remaine the right angled figure, made of the 2. opposite sides, AC. and BD. which right angled figure, if I diuide by the knowne side, BD. the quotient shall produce the side vnknowne, AC. being the sine of the difference of the two vnequall arches giuen, AB. and BC. which was also it selfe to be demonstrated.

Example



Example of both Members.

Let the greater arche, A B. be ——— 20. deg.
The lesser arche, B C. be ——— 15. deg.

The summe of these 2. arches, shalbe 35. deg.
Their difference ——— 05. deg.

The sines of the arches giuen, and of their complements are.
The sine of the arch, A B. is 3420201. the sine of the Complement A D. 9396926.
The sine of the arch, B C. is 2588190. the sine of the Complement C D. 9659258.

Then let them be multiplied alternatly, the sine of the arch, A B. by the sine of the complement of the arch, B C. to wit, by C D. And the sine of the arch, B C. by the sine of the complement of the arch, A B. that is by A D. And

The greater product shalbe 3303660. } 3870858.
The lesser product shalbe —— 2432102. } 9905740.

The some diuided by ℣ *Radius* 5735763. } the sine of ½ some 35
The difference diuid. by *Radius*, 8711557. } the sine of ½ diffe. 5.

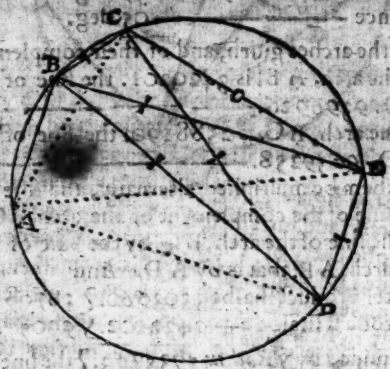
The sixth Problem.

38. *The sines of 2. vnequall arches, being giuen together with the sines of their Complements: to finde the sine of the complement of the total, or the sine of the complement of the difference of those arches.*

The

The Rule. Multiplie the sine of the one arche by the sine of the other arche. And also the sine of the Complement of the one arch, by the sine of the complement of the other arch, this being done: If you take the lesser produkt from the greater and diuide the Remainder by the Radius, you shall haue the sine of the complement of the somme of the 2. giuen arches. But if you add the 2. Produkts together: and diuide that totall by the Radius, you shall haue the sine of the complement, of the difference of the 2. giuen arches.

The reuson of the rule. And this Rule hath 2. Members. The first is thus demonstrated. Let the sine of the greater arch, be the right line, A B, or D E. equall thereunto; the sine of the complement, the right line, B E, the sine of the lesser arch, the right line B C. the sine of the complement, the right line, C D. the sine of the some, the right line A C. the sine of the complement of the somme, the right line C E. the *Radius*, the right line, B D. Then because the figure B C E D. is a quadrilater all figure inscribed in a Circle, and intersected with diagonals. Therefore if I multiplie the sine of the complement of the arch, A B. to wit, the right line, B E. by the sine of the complement of the arch,

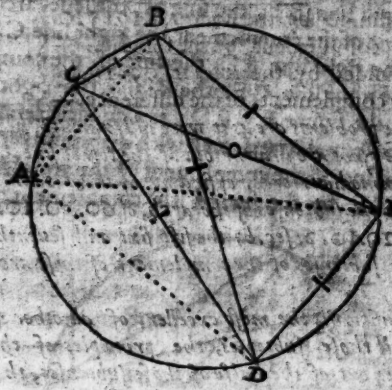


B C. to wit, by the right line C D. I shall haue a right angled figure, of the diagonals equall to 2. right angled figures, made of the 2. opposite sides, by the 54. of the first. Then againe if I multiplie the sine of the arch, B C. by the opposite sine of the arch, D E. which is equall to the arch, A B. and substract this

right

right angled figure, from the right angled figure, of the diagonals, the Remainder shalbe the right angled figure, made of the 2. other opposite sides, B D. and C E. which right angled figure if I diuide by the knowne side, to wit, by the *Radius*, *B D*. the quotient shall produce the vnknowne side, C E. the sine of the complement of the some of the 2. giuen arches, *A B*. and *B C*. which was to be demonstrated.

The latter member is thus demonstrated. Againe let the sine of the greater arch, be the right line, *A B*. or E D. the sine of the complement, the right line *B E*. the sine of the lesser arch, *B C*. the sine of the complement, *C D*. the sine of the difference, *A C*. the sine of the complement of the difference, *C E*. the *Radius*, *B D*.



Then because the figure *C B E D*. is a quadrilaterall figure, inscribed in a circle, and intersected with diagonall lines. Therefore if I multiplie euery of the 2. opposite sides to wit, the line *B C*. by the line D E. and the sine of the complement, *B E*. by the sine of the complement C D. and adde their products together, I shall haue the right angled figure, made of the diagonall, *C E*. and *B D*. which if I diuide by the knowne side, to wit, by the *Radius*, *B D*. the quotient shall produce the vnknowne side, *C E*. the sine of the complement, of the difference of the 2. giuen arches, *A B*. and *B C*. to wit, the sine of the complement of the arch, *A C*. which also was to be demonstrated.

Example

Example of both Members.

Againe let the greater arch, *A B.* be 20. deg.
The leſſer arch, *B C.* be ——— 15. deg.

The ſomme or totall is ——— 35. deg.
The difference is ——— 5. deg.

The ſines of the giuen arches, & their complements as before.
The ſine *A B.* 3420201. The ſine *A D.* or *B E.* 9396926.
The ſine *B C.* 2588190. The ſine *C D.* ——— 9659258.

Let *A B.* be multiplied by *B C.* and *B E.* by *C D.* and the products ſubſtracted from, and added one to another. And

The greater product ſhall be 90767335 } 2640908.
The leſſer product ſhall be — 88521350 } 2026190.

The difference diuided by the *Radius*, 8191520. }
The ſine of the complement, of the ſomme. — }
The ſomme diuided by the *Radius*, 9961946. }
The ſine of the complement, of the difference — }

39 *Theſe nine problems are as it were inſtruments, by whoſe help all the reſt of the ſines are drawne out of the totall ſine.*

The moſt commodious order of finding of them is thus; firſt are to be found out the ſubtenſes of the arch, of 60. 30. 10. 2. 1. deg. 20. 10. 2. 1. *m.* 20. 10. 2. *ſec. by the fift ſixt and ſeuenth problems.*

Likewiſe the ſubtenſes of the complement of thoſe arches, by the firſt problem.

For this inquiſition is the moſt excellent of all other: that you may right y call thoſe ſubtenſes, the principles of the Canon of Triangles. Then out of the halfe of thoſe ſubtenſes, that is, out of the ſines of the arches of 30. 15. 5. 1. *d.* 30. 10. 5. 1. *m.* 30. 10. 5. 1 *ſec. together with the ſines of the complements of thoſe arches, you ſhall eaſilie finde out all the reſt of the ſines, by the ſecond* 8. *and* 9. *Problems. By the ſecond Problem, by finding out the ſine, of* 2. *d. alſo of* 2. *m. and of* 2. *or* 20. *ſec. By the* 8. *and* 9. *by continually adding to the ſines hitherto found, the ſine of* 1. *de. or of* 10. *ſec. or alſo of one ſecond: as you would haue the table breife, or more ample.*

I found out the ſubtenſes aforeſaid, of 60. 30. 15. &c. after the ſame manner, as I haue ſet downe in the partes of the *Radius*, of 10000000000000000000000000000. And ſo I found as followeth.

The

The Arches.	The Subtenses.				
60.de. 00'.00"	100000	00000	00000	00000	00000
30. 00. 00	51763	80902	05041	52469	77977
10. 00. 00	17431	14854	95316	34711	61285
2. 00. 00	3490	48128	74567	02563	88379
1. 00. 00	1745	30709	96747	86992	97769
20. 00	581	77559	68723	86884	86923
10. 00	290	88810	61083	07015	25490
2. 00	58	17764	09126	84919	27486
1. 00	29	08882	07640	14734	29548
20	9	69627	36183	92296	70823
10	4	84813	68106	20557	94030
2	0	96962	73622	15278	56399

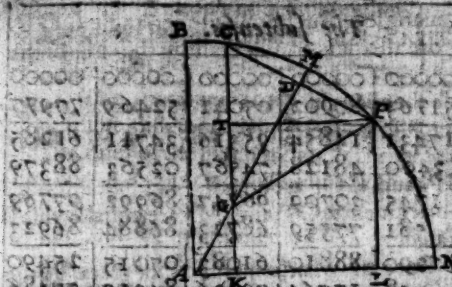
The halfe of the arches.	The halfe of the subtenses.				
30.de. 00'.00"	50000	00000	00000	00000	00000
15. 00. 00	25881	90451	02520	76234	88988
5. 00. 00	8715	57427	47658	17355	80642
1. 00. 00	1745	24064	37283	51281	94189
30. —	872	65354	98373	93496	48884
10. —	290	88779	84361	93442	43461
5. —	145	44405	30541	53507	62745
1. —	029	08882	04563	42459	63743
00. 30	014	54441	03820	07367	14774
10	4	84813	68091	96148	35411
5	2	42406	84053	10278	52015
1	0	48481	36811	07636	78199

40 In this *Theorem*, is an excellent Compendium, of the sines.

The difference of the sines, of 2. Arches, equally distant from 60. degrees, is equall to the sine of the distance.

The declaration. Let C N. and P N. be the 2. arches, equally distant from 60. degrees, M N. that is equally distant from the point, M. And let the right lines C K. and P L. be the sines of

 those



those arches, being perpendiculers vpon the right line, A N. by the 3. coniect. of the 7. And thereupon parallell one to another by the 38. of the first.

Moreouer let the right line, P T. be drawne perpendiculer vpon the right line, C K. parallell to the right line, K L. by the 38. of the first. This right line, L P. cutteth from the right line, C K. another line, T K. equall vnto P L. by the 39. of the first, And leaueth the right line, T C. for the difference of the sines C K. & P L.

Lastlie the sines of the distance, of either of them from 60. d. Let be the right line, C D. or D P. I say that the right line, T C. is equall to the right line, C D. or D P.

The demonstration. For because in the Triangle, G C P. that the perpendiculer G D. doth bisect the base C P. by the 12. hereof, and by the pro: Therefore the sides G C. and G P. are equall by the 23. of the first; And the angles, C G D. and D G P. are also equall by the same, and lastly the angles, *G C P.* and *G P C.* are likewise equall, by the 20. of the first. But the angle *C G D.* is 30. d. for that it is equall to the angle, *B A M.* by the 38. of the first.

Therefore the angle, *C G P.* is 60. d. for that it is double to the angle C G D.

But because the angle C G P. is 60. d. therefore the other two angles, G C P. and G P C. added together, are 120. d. by the 49. of the first.

But these other 2. are demonstrated to be equall, therefore euery of them is 60. d.

And the angle C G P. is also so many degrees, therefore the triangle

triangle C G P. is equiangled. But becauſe the triangle C G P. is equiangled, therefore alſo it is equilaterall by the 28. of the firſt.

Moreouer, becauſe the triangle C G P. is equilaterall, therefore the perpendiculer P T. b-ſecteth the baſe C G. by the 23. of the 1.

Then the ſides C P. and C G. are equall.

Therefore alſo their biſegments, C T. and C D. are equall, which was to be demonſtrated.

Conſectaria. The ſines of whatſoeuer 60. degrees being giuen, you may finde the ſines of the other, 30. degrees by addition or ſubſtraction only.

The illuſtration by Numbers. Let the arches C N. be 70. d. P N. 50 d. C M. or P M. 10. d. for ſo many degrees are the arches of 70. d. and 50. d. diſtant from the arch, of 60. degrees. And let firſt the ſines of 70. d. and 10. d. be giuen; And let the ſine of 50. d. be demaunded.

From the ſine of 70. d. C K. —— 9396926
Subſtract the ſine of 10. d. C D. or C I. —— 1736482

The remainer wilbe the ſine of 50. d. T K. or P L. 7660444

Then let the ſines of 70. d. and 50. d. be giuen. And let the ſine of 10. d. be demaunded.

From the ſine of 70. deg. C K. —— 9396926
Subſtract the ſine of 50. deg. T K. or P L. —— 7660444

The remainder will ye the ſine of 10 d. T C. or C D. 1736482

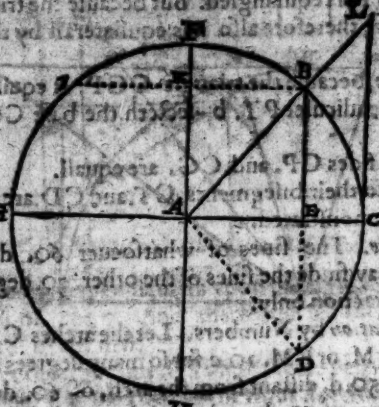
Laſtly, let the ſines of 50 deg. and 10 deg. be giuen, and let the ſine of 70. deg. be demaunded.

To the ſine of 50 deg. P L. or T K. —— 7660444
Adde the ſine of 10 deg. D P. or T C. —— 1736482

The whole will be the ſine of 70. deg, C K. —— 9396926

41 *And thus farre of the making of the tables of right ſines. The tables of verſed ſines are not needfull, as aforeſaid.*

42 *The tables of Tangents and ſecants, are thus made out of the tables of right ſines.*



1 As the sine of the complement to the sine of an arch: So is the *Radius* to the Tangent, of that arch.

2 As the sine of the complement to the *Radius*: So is the *Radius* to the *Secant*, of that arch.

For by the 46. of the 1.

1 As AE. to EB. So is AC. to CL.

2 As AE. to AB. So is AC. to AL. As for example: Let the *Tangent* and *Secant*, of the arch, BC. 30. d. be sought for: The sine of 30. d. is 5000000. BE. The sine of the complement, 60. d. is 8660254. AE. Then I say.

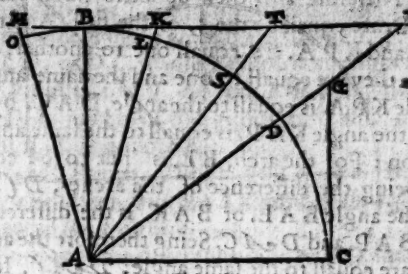
1 As AE. 8660254. to BE. 5000000. So is AC. 10000000. to CL. 5773503. Therefore the *Tangent*, of the arch, of 30. d. is 5773503.

2 As AE. 8660254. is to AB. 10000000. So is AC. 10000000. to AL. 11547005. Therefore the *Secant* of the arch of 30. d. is 11547005.

43 *The breife Rules of the* Tangents *and* Secants, *are excellent in these* 3. *Theorems following.*

The first Theorem. The difference of the Tangents, *of any* 2. *arches, making a Quadrant, are double to the* Tangent, *of the difference of those arches.*

The



The declaration. Let the 2. arches making a quadrant be C D. and B D. whoſe *Tangents*, are C G. and B P.

Let B S. be an arch, made equall to C D. whereupon S D. will be the arch of the difference, of the 2. giuen arches, C D. or B S. and B D. Alſo let the *Tangent*, B T. be equall to the *Tangent*, C G. whereupon the right line, T P. wilbe the difference of the *Tangents* giuen C G. or B T. and B P. Laſtlie let the arches B L. and B O. (whoſe *Tangents* are B K. and B M.) be made equal to the arch S D. I ſay that the right line T P. being the difference of the 2. giuen *Tangents*, C G. and B P. is double to the right line, B K. being the *Tangent* of the difference of the 2. giuen arches: Or, which is all one, I ſay that the right line, T P. is equall to the right line, M K.

The demonſtration. For if you take equall things from equall, the remainder ſhall be equall. But the right lines, K P. and M T. are equall. Therefore if you take the right line, K T. from both of them, the right T P. and M K. remaining, ſhall be equall.

The aſſumption is prooued. For thoſe things that are equall to one and the ſame thing, are alſo equall one to another.

But the right lines, K P. and M T. are equall to the ſame right line, K A. Therefore they are equall one to another.

Againe, the aſſumption is prooued: And firſt that the right line, K P. is equall to the right K A. is thus prooued.

Becauſe the angles K A P. and K P A. in the triangle, A K P. are equall. Therefore alſo their oppoſite ſides, that is the ſide K A. and K P. are equall by the 5. of the firſt. And that the angles, K A P. and K P A. are equall one to another, thus appeareth, for that they are equall to one and the ſame angle, D A C. For the angle K P A. is equall to the angle, D A C. by the 38. of the 1. And the angle K A P. is equall to the ſame angle, D A C. by oppoſition: For the arch, B L. is put to be equall to the arch, S D. being the difference of the arches, *D C.* and B D. Therefore the angle B A L. or B A K. is the difference betwixt the angles, B A P. and *D A C.* Seing therefore the angles K A P. and K P A. are equall to the ſame angle, *D A C.* It followeth neceſſarilie that they are alſo equall one to another.

Then that the right line, *M T.* is equall to the right line, K A. or to the right line, M A. by the propoſition, is thus prooued.

For that in the triangle, *A M T.* the angles, *M T A.* and *M A T.* are equall, therefore alſo the ſides oppoſite vnto them that is, the ſides *M T.* and *M A.* are equall by the 5. of the 1.

And that the angles, *M T A.* and *M A T.* are equall, thus appeareth. Becauſe the angle *M T A.* is equall to the angle *T A C.* by the 38. of the 1. And the angle *T A C.* is equall to the angle, *T A M.* by the propoſ: for the arches, *C S.* and *S O.* are put to be equall. The ſame reaſon is, if the difference *B L.* be greater then halfe the complement, B S. onely the letters L. and S. and alſo K. and *T.* are put one for another.

Generally therefore the difference of the *Tangents* of two arches, making a quadrant, is double to the *Tangent*, of the difference of thoſe arches which was to be demonſtrated.

Conſectaris. Therefore the *Tangents*, of 2. arches, being giuen, making a quadrant, the *Tangent* alſo of the difference of thoſe 2. arches, is alſo giuen.

And contrarilie, the *Tangent* of the difference of theſe 2. arches, being giuen, together with the *Tangent* of the one arche; The *Tangent* of the other arch, is alſo giuen.

The appendix.

This *Theorem* may thus be propounded. The double *Tangent* of an arch, with the *Tangent* of halfe the complement, is equall to the *Tangent* of the arch, compoſed of the arch giuen, and halfe

halfe the complement thereof.

For if the arch, B L. be put for the arch giuen, the double *Tangent* thereof shall be *T P.* by the demonstration before going. And the complement of the arch, B L. shall be *L C.* whose halfe is the arch, L D. or *D C.* whose *Tangent* is the right line, *G C.* or B T. But *T P.* added to *B T.* maketh *B P.* being the *Tangent* of the arch *B D.* composed of the giuen arch, *B L.* and halfe the complement L D. Therefore &c.

44 *The second Theorem. The* Tangent *of the difference of 2. arches, making a quadrant, with the* Tangent *of the lesser arch, maketh the* Secant *of the difference.*

As the *Tangent* of the difference, B L. or B G. that is the right line *B K.* or *B M.* with the *Tangent* of the lesser arch, *D C.* or *B S.* that is with the right line, *B T.* maketh the right line, *M T.* which is equall to the right line, *A K.* being the *Secant* of the difference *B L.* by the demonstration of the first *Theorem.*

Consectarie. Therefore the *Tangent* of the difference of 2. arches, making a quadrant, and the *Tangent* of the lesser arche being giuen, the *Secant* of the difference is also giuen. And contrarilie &c.

The Appendix.

This *Theorem* may be also thus propounded. *The* Tangent *of an arch, with the* Tangent *of halfe the complement, is equall to the* Secant *of that arch.* For if you haue the arch, *B L.* or B O. for the arch giuen, the *Tangent* of the arch giuen, shall be B M. the *Tangent* of halfe the complement, shall be B T. which 2. *tangents* added together, make the right line, M T. But the right line M T. is equall to the right line, *A K.* by the demonstration of the first *Theorem,* which right line, *A K.* is the *Secant* of the arch giuen B L. by the propo: Therefore &c.

45 The 3. Theorem. *The Tangent, of the difference of 2. arches, making a quadrant with your Secant of their difference, is equal to the tangent, of the greater arche.*

As the tangent of the arch, B L. being the difference of the 2. arches, B C. and D C. making a quadrant, with the *Secant* of the same arch, B L. that is the right line, B K. with the right line A K. is equall to the right line, B P. by the demonstration of the first *Theorem.*

Corollarie. Therefore the *Tangent*, of the difference of two arches, making a quadrant being giuen, with the *Secant*, of their difference, the *Tangent* of the greater arch is alſo giuen: And contrarilie &c.

The Appendix.

This *Theorem*, may alſo be thus propounded. *The Tangent of an arche, with the Secant thereof, is equall to the Tangent of an arche, compoſed of the arch giuen, and halfe the complement thereof.*

For, if you haue the arch, B L. for the arch giuen, B K. ſhall be the *Tangent*, and A K. the *Secant* of the arch giuen.

But the right lines, A K. and K P. are equall by the demonſtration of the firſt *Theorem*, ; Therefore the *Tangent*, of the arche giuen, B L. that is the right line, B K. with the *Secant* of the ſame arch, that is A K. is equall to the right line, B P. which is the *Tangent* of the arch, B D. being compoſed of the giuen arch, B L. and $\frac{1}{2}$ L D. being $\frac{1}{2}$ the complement.

46 *In the table following, you haue examples of the three precedent Theorems.*

Deg. mi.	Tan. of 30″. is 1454	The Secants.
89. 59	34377466738	34377468192
1	2909	
	34377463829	4 By subſtr.
89. 58	17188731914	17188734823 By addi.
2	5818	
	17188726096	
89. 56	8594363048	8594368866
4	11636	
	8594351413	2 By subſtr.
89. 54	4297175706	4297187341 By addi.
6	23271	
	4297152435	9 By subſtr.
89. 44	2148571217	2148599488 By addi.
16	46542	
	2148529675	80 By subſtr.
89. 28	1074264817	1074311379 By addi.
32	93087	
	1074171750	
88. 56	537085875	537178962
1. 04	186190	
	536899685	3 By subſtr.
87. 52	268449842	268636032 By addi.
2. 08	372509	
	268077333	6 By subſtr.
85. 44	134038666	134411175 By addi.
4. 16	746053	
	133292613	60 By subſtr.
81. 28	66646305	67392359 By addi.
8. 32	1500458	
	65145848	
72. 56	32572924	34073382
17. 04	3700034	
	29502890	
55. 52	14751445	17821479
34. 08	6778997	
	7972448	
21. 44	3986224	10765221
68. 16		

In which

In which table, first the *Tangents* of two arches, making a quadrant being giuen, to wit, of the arch of 89 deg. 59'. and of the arch of 1'. The tangent of their difference, being 89 d. 58'. is found out. Moreouer, by this tangent, and the tangent of the complement of the arch being 2'. the tangent of the difference being 89 deg. 56'. is found, And so forward, vntill the complement vnder written, could not be taken out of the arch giuen any more, which was done vnto the arch of 29 d. 44'. whose complement is 68 deg. 16'. which cannot be taken from the arch 21 deg. 44". So then I say that all the tangents are sought out by the first Theorem. Then all the Secants of the same arches, except the first, are found out, by continuall adding of the tangent of the difference to the tangent of the lesser arch, by to second Theorem: or by substracting the tangent of the difference from the tangent of the greater arch by the third Theorem. And the first Secant is found, by adding the tangent of ½ the complement, to wit, 30". to the tangent of the giuen arch being 89 deg. 1'. by the second Theorem.

But if, beyond this continuation of examples, viz. by the tangent of the arch giuen, being 21 deg. 44'. and of the complement thereof being 34 deg. 8'. the tangent of the arch composed of the giuen arch, 21 deg. 44'. and the complement being 34 deg. 8'. to wit, the arch of 55 deg. 52'. be demaunded: The appendix of the first Theorem is to be vsed.

And if by the same tangents of the arch of 21 deg. 44'. and of halfe the complement 34 deg. 8'. the secant of the arch of 21 deg 44 were demaunded: You must vse the appendex of the second Theorem.

Lastly, if by the tangent, & secant of the arch of 21 deg. 44'. the tangent of the arch composed of the arch giuen, being 21 d. 44'. and ½ the complement being 34 deg. 8'. to wit, of the arch 55 deg. 52'. were demaunded: Then you must worke by the Appendex of the third Theorem.

47 *And thus much of the making of the table: Now the examination*

mination of the tables made, may be done diuers wayes, viz. either by the rules and precepts hetherto set downe for the making of the tables, or by the differences, seconds, thirds, &c. of the Sines, Tangents and Secants.

48 *And by what meanes this proofe may be made: it is to be vnderstood, that howbeit some Number in the end, may seeme to be a false Number, yet it is not a false Number.* As if you examine the tangents following by the 43. hereof, after this manner.

77 deg. 49'. the tangent.	45045072.	
12. 31. the Tangent	2219999	
		42825073
64 deg. 55'. the Tangent.	21412536	

The last tangent 21412536. in the last figure, doth not answere to the tangent put in the table, for there the last figure is 7. And yet there is no error in these 3. tangents: And the reason why the tangent 21412536. came out in this example lesse iust by 1. is, because the tangent 2219999. was greater, iust by 1. and therefore it is substracted too much from the tangent 45045072. But if you put the tangent 2219998. for the tangent 2219999. this shall be lesser then the true tangent, and the last tangent 21412537. shal come for the greater then the truth. Therefore for such small difference, which by no meanes can be avoyded, the table is not to be accompted erronious.

49 *In the other figures, except the last, if any error be, It may easily be found out by the first, second, and third differences.* At aduenture in the table, let the tangent of 77 deg. 26'. be taken, which is 44494381. And let there be suspect that there is some error therein. Set downe in order some of the tangents with their differences, first, second and third, after this manner:

deg.mi.	Tangents.	differ. 1.	diff. 2.	diff. 3	deg.mi
12. 31.	45074072				77. 29
12. 32.	44983221	61851			77. 28
12. 33.	44921532	61689	162		77. 27
12. 34.	44860004	61528	161	1	77. 26
12. 35.	44798636	61368	160	1	77. 25
— 36	44737428	61208	160	0	— 24
37	44676379	61049	159	1	— 23
38	44615489	60890	159	0	— 22
39	44554756	60733	157	2	— 21
40	44494381	60375	358	0—1	— 20
41	44433762	60419	156	2	— 19
42	44373499	60263	156	0	— 18
43	44313392	60107	156	0	— 17

And you ſhall partly perceaue either by the firſt, or certainly by the ſecond differences, that the Number 4494381. in the third place, from the right hand is falſe, becauſe in the ſecond differences after 157. followeth 358. which cannot be but falſe, Therefore put 158. for 538. and ſubſtract that 158. from the firſt difference next aforegoing, being 60733. the remainder ſhall be in the firſt differences 60573. for the ſum next following: which againe if you ſubſtract from the tangent aforegoing 4454756. the Remainder ſhall be 44494181. for the tangent deſired: And ſo the error will be amended, and the Numbers ſtand thus, following one after another.

—39	44554756	60733	157	2
40—	44494181	60575	158	1
41—	44433762	60419	156	2
42—	44373499	60263	156	0
43—	44313392	60107	156	0

50 *Some men haue ordered their tables in an other forme. But this which you ſee ſeemeth to me moſt conuenient; Wherein the Sines, Tangents and Secants, of the arches leſſe then a halfe Quadrant, are placed in the left ſide: But the Sines, Tangents and Secants, more then halfe a Quadrant, are placed in the right ſide, to the end that whether*

whether the queſtion be of an arch more or leſſe then a Semi-quadrant, you may preſently finde the complement thereof. And the Sines, Tangents and Secants, of the arches leſſe then a Semi-quadrant, together with their arches increaſing downwards. But the Sines, Tangents and Secants, of the arches greater then a ſemi-quadrant, together with their arches do increaſe aſcending vpwards by euery minute, except in the firſt degree and the Complement thereof, where I haue alſo vſed one, two, or ten ſeconds, becauſe otherwiſe the Calculation there in ſeconds, could not haue bene without error. In ſtead of the differences, I haue put the proportional part either of minuts or of tenths of ſeconds, for the more eaſe in making the tables. I haue alſo added the increaſe, wherein the vnequall proportionall parts, doe increaſe either by euery one, or by euery ten ſeconds, for the greater preciſenes.

I haue taken diuers Radiuſſes for neceſſitie, to wit, of 5. 7. 8. 9. 10. 11. or 12. figures; Which variety, the skilfull Arithmetician will eaſily reconcile, by vſing the Radius for his worke of ſuch magnitude, as euery Number ſet downe in the table, may anſwere thereunto. Which that it may preſently appeare, I haue euery where diſtinguiſhed with a point put betwixt, the Sines, Tangents and Secants, made for the Radius 100000. from the reſt, greater then that Radius; Nay where the Radius is more then 10. figures, I haue put 2. points betwixt, whereby the Sines, Tangents and Secants, of the Radius of 10. figures, may by a marke be diſcerned, and knowne from the greater Sines, Tangents and Secants: Where no Number is after the point, there the Radius is only of 5. Ciphers 100000. as in all Tangents and Secants, of the laſt fiue degrees.

51 The vſe of this table generally is thus: That you may readily finde out the Sine, Tangent, and Secant of any arch or angle giuen, not exceeding 90. deg. together with the Sine, Tangent, and Secant of the Complement: or contrarily by the ſame tables, the arch of any ſine, tangent, or ſecant giuen. And ſo in the working of triangles, you may proceed without delay. As if you would haue the Sine, Tangent and Secant, of the arch or angle of 30. deg. or of the complement thereof: All theſe will be giuen you, in the tables according to the Radius. 10000000.

Of the arch of 30. deg.		*Of the Complement.*	
The Sines.	5000000	The Sine is	8660254
The Tangent is	5773503	The Tangent is	17320508
The Secant is	11547005	The Secant is	20000000

If 5773503 be a Tangent giuen, and it be demaunded what arch or angle answereth thereunto: The table will shew that the arch or angle answering to that tangent, is 30. de. And likewise in the other side of the table it will shew you, that the arche or angle of 60. deg. is the Complement thereof.

52 *But if peraduenture seconds, be adioyned to the minutes, and that you must vse them in the worke, then proceed as the examples following shall teach you.*

The first example. If the sine of 12. deg. 6'. 23". be to be found, Take in the beginning of the tables the sine of 12 deg. 6'. which is 2096186. Then gather by the proportionall parts how much the remainder 23." will require: in saying,

10" giues 474. parts, what shall 23".

23

1422

948

1090|2

the sa: is 1090. parts.

Lastly, to the former giuen sine. —— 2096186
Adde the proportionall part now found. — 1090

And you shall haue the sine. —— 2097276. desired.

The second Example. If the tangent of the arch of 88 degrees 51' 34" be demaunded: First take out of the tables the tangent of 88. deg. 51" which tangent according to the Radius 100000. is 4981573. Then you shall finde the proportionall part, for 34". after this manner.

10.

10" —— —— ——	12065. A.	
the increaſe. ——	000 59	This ad continually firſt to A. then to B, thirdly to C & you ſhall haue ABC for 30" then if you multiply D. by 4. & cut of the laſt cipher, you ſhall haue 4897. for the other 4" Now ad ABCE. together, and you ſhall finde F. w is the tangent of 34", for the tangent required.
10" —— —— ——	12124 B.	
10" —— —— ——	12183 C.	
10". —— —— ——	12242 D	
4 —— —— ——	4897 E.	
34 min. —— —— ——	41269 F	
Ad the tangent of 88d. 51'.	4981573	
The totall is —— ——	5022842.	

The third Example. If the tangent of 89. deg. 39'. 24" were to be found. You muſt thus proceed:

The tangent of 89 deg. 39' 20". is 16634058. A.

1". is	13425 B	
The increaſe is	00042	This ad continually to B C. & D. that you may finde CD & E. Laſtly, colle& ABCDE. into 1. ſome for the tangent deſired.
1". is	13447 C	
1". is	13469 D	
1". is	13491 E	
And you ſhall haue 89. 39. 24".	16687890	

Or more briefly, multiply the proportionall part of 1. *ſecond by* 4. *and the increaſe by ſo many vnites as are in the progreſſion of* 4. *places, that is by* 6. *(for ſuch is the progreſſion of* 4. *places* 0. 1. 2. 3 *min. which progreſſion are* 6. *vnites) and you ſhall haue the ſame Tangent, after this manner.*

The tangent is ——	16634058 A.	
1". is ——	13425 which	
Multiply by 4. is —	53700 B.	
The increaſe —	00042 which multiply	
By 6. maketh —	00132 C. Adde ABC.	
Together, & you ſhall haue —	16687890. for the tangent deſired.	

53 *But if contrarily any sine, tangent or secant were giuen, whose arch you would also finde in seconds precisely. So proceed as the examples following will teach you.*

The first example. If 2097276. were giuen for a sine, the Radius being 10000000. And it were demaunded what arch were answerable thereunto?

First, seeke out in the tables, the next lesser sine, and substract that from the sine giuen, and note the arch agreeable thereunto: Then out of the remainder you shall collect the seconds after this manner.

The sine giuen is — — 2097276
The lesser sine next vnto it, is — 2096186 of ỹ arch 12d.6″

which substracted, the Remainder is 1090
10″. in the table is answerable to — 474
Now if 474. giue 10″. what shall — 1090 giue?

10900
474 (2
948
1420
474 (3 almost
1422

Therefore the arch sought of, is
12 deg. 6′. 23″. almost.

The second example.

If 5022839, according to the Radius 100000. be giuen, And the arch answering therevnto were demaunded: First againe finde in the tables the next lesser tangent, and the arch answering thereunto. Then substract that lesser tangent, from the tangent giuen, and out of the Remainder you shall gather the seconds after this manner.

The tangent giuen, is 5022839
The lesser tangent next 4981573 of the arch of 88. d, 51′

The Remainder — — 41266
Substract — 12065 the parts for 10″.

The Remainder is — 29201 From whence

10″

10". giues — 12065 } 12124 the parts for 10" ſubſtracted
The increaſe 00059 } 17077 remaineth: from whence
10", — 12124 } 12183 the parts of 10" ſubſtracted
10", — 12183 } 4894 } remaineth
10", — 12242 } 1224 } the parts for 01". Now 1224
1 ", — 1224 } 4896 } is in 4894 almoſt 4 times.

for foure times 1224 maketh 4896: Therefore the arch anſwerable to the tangent giuen, is 88 deg. 51'. 34".

The third Example.

If the tangent 16687890. be giuen, according to the Radius 100000. And it be demaunded what arche is anſwerable thereunto: You ſhall proceed in this order.

The tangent giuen, is 16687890
The next leſſer tangent 16634058 of the arch 89 deg. 39' 20"
which ſubſtracted Reſteth 53832

The parts of 1', is — 13425 A.
The increaſe is ——— 00022 this adde to A.B. and C.
13447 B.
13469 C.
13491 D. Now adde A.B,C.&D

The totall amounts to — 53832 for 4" Therefore the arch anſwering to the tangent giuen, is 89 deg. 39', 24".

54 *By this Table, after this manner, you ſhall be able without any error in the doctrine of Triangles, to worke to ſeconds. And in the firſt and laſt degree eſpecially, more certainely then by* Rhæticus *his great tables: But in all other degrees,* Rhæticus *his tables are better; For by that you ſhall worke more ſpeedily, & not only to ſeconds, but alſo you may worke by it to thirds and fourths exactly. Therefore if you be wiſe and of ability, be not without that Table.*

The end of the ſecond Booke.

THE THIRD BOOKE OF TRIGONOMETRIA.

By *B.P.*

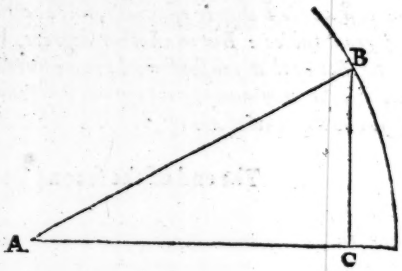
Of the dimension of plaine Triangles.

HEtherto I haue treated of the principles of Trigonometria, and of the neceſſarie tables of Sines, Tangents and Secants, for the exerciſe thereof. Now foloweth that Trigonometrie it ſelfe, or the meaſuring of Triangles, as well plaine as Spharicall. In the explaining of both which, becauſe there is no need of any other rule then the rule of proportions, as is aforeſaid; Firſt, I will ſet downe certaine Axiomes, *whereby may be underſtood what proportions are in certaine triangles, and parties of Triangles: Which* Axiomes, *therefore I will call the* Axiomes *of proportions. Then I will ſhew how thoſe* Axiomes *are to be vſed, or how by helpe of a few of thoſe* Axiomes *euery demaund, in any Triangle propounded by what ſoeuer there termes giuen; may be quickly found out.*

The Axiomes of proportions that are plaine Triangles are chiefly 4. being ſufficient enough for euery reſolution of any of them, beſides that golden foundation of all Trigonometrie, which I haue explained in the firſt booke the 46. Propoſition.

The firſt Axiom.

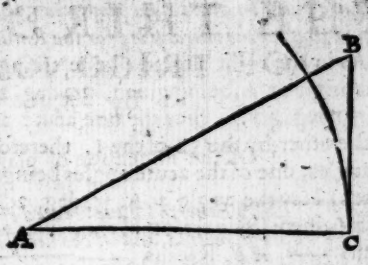
In all plaine right angled Triangles, euery ſide may be put for the Radius, agreeable to the doctrine of Triangles: For if you put the ſide ſubtending the right angle for Radius; the ſides including the right angle are ſines of the accute angles oppoſite vnto them.



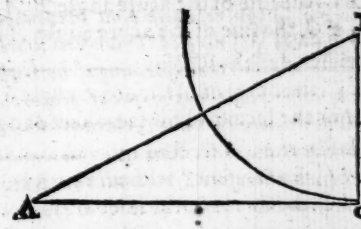
If you put for Radius the greater of the

ſides

ſides including the right angle, the leſſer of the ſides including the right angle is the Tangent, and the ſubtendent line to the right angle is the Secant of the leſſer accute Angle.



If you put the leſſer of the two ſides including the right angle for Radius: the greater ſide including the right angle is the Tangent, and the ſubtendent of the right angle is the Secant of the greater accute angle.



As in the plaine triangle A B C. If you put the ſide A B. ſubtending the right angle for Radius, the leſſer ſide B C. including the right angle, is the ſine of the leſſer oppoſite angle B A C. and the greater ſide including the right angle, is the ſine of the greater oppoſite accute angle A B C.

But if you put A C. the greater ſide including the right angle for the Radius, the leſſer ſide B C. including the right angle, is the tangent of B A C. the leſſer acute angle oppoſite, and the ſubtendent *A B.* is the ſecant of the ſame acute angle.

Laſtly, if you put the leſſer ſide *B C.* including the right angle for Radius, the greater ſide *A C.* including the right angle, is the tangent of *A B C.* the greater acute angle oppoſite, and the ſubtendent *A B.* is the ſecant of the ſame acute angle.

The firſt Conſectarie.

Therefore in all right-angled plaine Triangles, the angles being giuen, the reaſon of the ſides are alſo giuen, three wayes. And conſequently.

One ſide being giuen beſides the three angles, euery of the other

ſides are giuen by a threefold proportion, that is whether you ſhall put this, or that, or the third ſide, for the Radius.

As in the right angled plaine triangle, propounded *ABC.* The angle at A. being giuen, 30 deg. 20′. and ſo the angle at *B.* 59 deg. 40′. (For the one acute angle is the complement of the other, by the 32. of the 1. therefore in plaine right angled triangles, one of the acute angles being giuen, all the angles are giuen) I ſay the angle at A. 30 deg. 20′. and at *B.* 59 deg. 40′ being giuen; The reaſon of their ſides are giuen, either

Thus —— *AB.* Radius ——— 10000000
BC. the ſine of the acute angle *BAC.* 5050298
AC. the ſine of the acute angle *ABC.* 8631019

Or thus *AC.* the Radius —— —— —— 10000000
BC. the tangent of the acute angle *BAC.* 5851335
AB. the ſecant of the ſame acute angle 11586118

Or laſtly thus, *BC.* Radius ——— 10000000
AC. the tangent of the acute angle *ABC* 17090116
AB. the ſecant of the ſame angle —— 19800810

That is:

It is manifeſted by the table of triangles, what proportion, (for examples ſake) the ſide *AB.* hath to *BC.* viz.

Either as *AB.* 10000000 to *BC.* —— 05050298
Or as —— *AB.* 11586118 to *BC.* —— 5851335
Or laſtly, as *AB* 19800810 to *BC.* —— 10000000 And ſo of the reſt.

Therefore beſides the angles being giuen after this manner, let the ſide AB. be giuen 24. *feete; If it be demaunded how many foote the ſide BC. is? I will ſay.*

Either as *AB.* Radius 10000000 to *BC.* the ſine 5050298
So is *AB.* the ſide 24. foote to *BC.* 12. $\frac{12\,07152}{10\,000000}$ foote.
Or as *AB.* the ſecant 11586118. is to *BC.* the tangent 5851335
So is the ſide *AB.* 24. foote to *BC.* 12. $\frac{13\,98624}{11\,586118}$ foote.
Or laſtly, as *AB.* & ſecãt of & Cõ: 19800810 to *BC.* Ra: 10000000
So is *AB* the ſide 24, foote to *BC.* 12. $\frac{23\,90280}{19\,800810}$ foote.

So if the ſame ſide AB. be giuen 24. *foote, and that it be demaunded how many foote the ſide AC. is. I will ſay*

Either as *AB.* Radius 10000000. to *AC.* the ſine 8631019.

So is

So is the side *AB*, 24. foote to *AC*. 20. $\frac{24}{19.3.1.8.}$ foote.
Or as *AB*. the secant 11586118. to *AC*. the Radius 10000000
So is *AB*. the side 24. foote to *AC*. 20. $\frac{24}{23.17.6.4.1}$ foote.
Or lastly, as *AB*. the secant of the Complment 19800810. to *AC*. the tangent 17090116.
So is *AB*. 24. foote to *AC*. 20. $\frac{24}{23.8.5.1.3}$ foote.

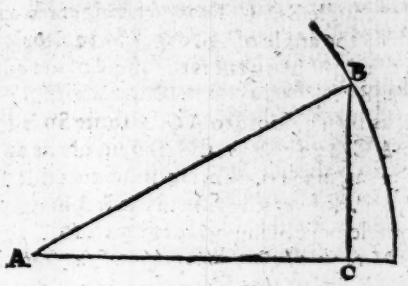
Likewise, if (the side AC. being given 20. $\frac{24}{19.3.1.8.}$ *foote) it be demaunded how many feete the side BC. is? I will say:*
Either as AC. the said 8631019. is to *BC*. (the sine 5050298.
So is the side *AC*. 20. $\frac{24}{19.3.1.8.}$ foot to *BC*. 12 $\frac{12}{11.7.5.3}$ foot
Or as *AC*, the Radius 10000000. to *BC*. the tangent 5851335
So is *AC*. 20. $\frac{24}{19.3.1.8.}$ foot, to *BC*. 12. $\frac{12}{11.7.6.0.}$ foote.
Or lastly, as *AC*. the tangent of the Complement. 17090116. to *BC*. Radius 10000000.
So is *AC*. 20. $\frac{24}{19.3.1.8.}$ foote to *BC*. 12. $\frac{12}{11.5.3.1.}$ foote.

Now the skilfull Arithmetician in the serious vse of Trigonometria in his calculation, will alwayes frame his proportion so, that he may haue the Radius in the first place, to auoide the troublesome paines of diuision.

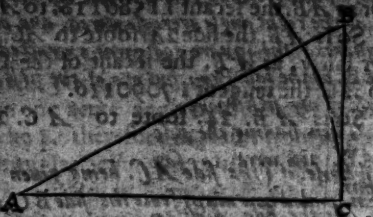
The second Consectarie.

Two sides whatsoeuer being giuen, both the acute angles is giuen by a double proportion, that is, as you put the one or the other of the giuen sides for the Radius.

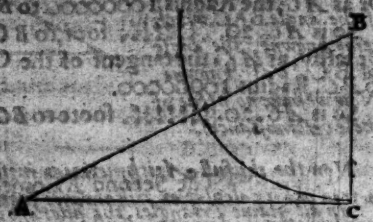
As in the plaine right angled triangle *ABC*. If the 2. sides *AB*. and *BC*. not including the right angle be giuen the one 5. and the other 3. And the 2. acute angles *A*. and *B*. are demaunded. I will say



Either as A B. 5. foote, to B C. 3. foote, So is AB. Radius 10000000 to the fine of the angle B AC. 6000000. which fine in the leaſt margent of the tables anſwereth to the angle B AC. 36d. 52′, 12″, & in the right margent is the fine of the complement being 53 deg. 7′, 48″.



Or as B C. 3. foote, to A B. 5. foote, So is BC. Rad. to AB. 16666666 the ſecant of the angle ABC. to which ſecant in the right margent of the tables, the angle of 53 de. 7′, 48″. anſwereth



And in the leaſt margēt is the angle of the complement, being 36 deg. 52′, 12″.

Likewiſe in the ſame right angled plaine triangle A B C. If the 2. ſides AC. and BC. be giuen, the one 4. and the other 3. foote: And the acute angles A. and B. be demaunded. I ſhall ſay

Either as AC. 4 foote to B C. 3. foote

So is A C. Radius 10000000. to B C. 7500000. the tangent of the angle B A C. To which tangent in the leaſt margent of the tables, the angle of 36. deg. 52′, 12″. for B A C. anſwereth. And in the right margent is the angle of the complement being 53 d. 7′, 48″.

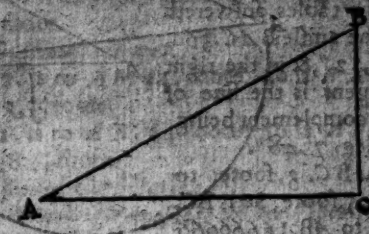
Or as B C. 3. foote to A C. 4. foote So is B C. Radius 10000000 to A C. 13333333. the tangent of the angle A B C. To which tangent, in the right margent of the tables, the angle ABC. 53 d. 7′, 48″. anſwereth: And in the leſt margent is the angle of the complement being 36. deg. 52′, 12″.

Note that before the tables of tangents were found out: the 2. ſides including the right angle being giuen; the acute angles, and the third

third side were thus found out. First the sides A C. and B C. including the right angle, were squared, and one of the somme of those 2. squares, the square roote was extracted: which roote was the side A B. by the last Pro: but one, of the first of Euclide: *that is, by the 50. of my first booke.*

Then hauing the side A B. you were to say thus.

As the side A B. to the side B C. So is A B Radius to B C. the sine of the angle B A C. which being knowne, the angle A B C. was knowne. Now we haue no neede of those circumstances.

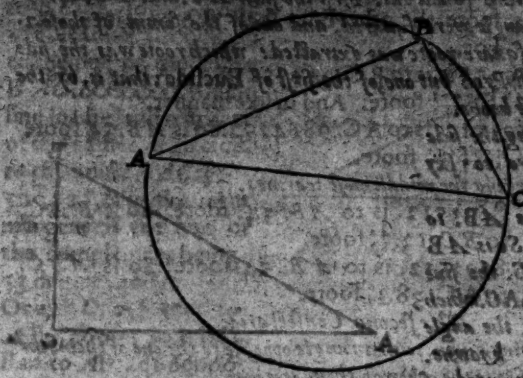


The Second Axiome.

In all plaine Triangles, the sides are in proportion one to another as the sines of the angles opposite to those sides.

For the sines are the one halfe of their subtenses: *But the sides of euery plaine Triangle are in proportion one to another, as the subtenses of the angles opposite to those sides.* Therefore also are the halfe of the subtenses in the same proportion. For the same reason, that is of the whole to the whole, the same reason is of the halfe to the halfe: As in the ninteenth Pro: of the second booke I haue manifested: And nature it selfe sheweth.

But that the sides of euery plaine triangle are in proportion one to another, as the subtenses of the angles opposite thereunto hence appeareth; Because a circle may be circumscribed about euery plaine Triangle, the Center being found out to the three points of ỹ three angles: which being done, the sides then themselues of that plaine triangle, are the subtenses of the angles opposite to those sides, that is, of the arches opposite to those angles, and are the double measures of them, by the 53. of the first.



As if the Circle, A B C. be circumſcribed about the triangle, A B C. the ſide A B. is made the ſubtenſe of the angle, A C B. that is of the arch, A B. which is oppoſite to the angle, A C B.

The ſide B C. is made the ſubtenſe of the angle, B A C. that is of the arch, B C. which is oppoſite to the angle, B A C. And laſtly, the ſide A C. is made the ſubtenſe of the angle, A B C. that is of the arch, A C. which is oppoſite to the angle, A B C.

Therefore the ſide A B. is in proportion to the ſide, B C. as the ſubtenſe of the angle, A C B. to the ſubtenſe of the angle, B A C. &c. which was to be demonſtrated.

The fifth Conſectarie.

Therefore the angles being giuen, the reaſon of the ſides is giuen; And conſequentlie.

One ſide being giuen, beſides the angles, euerie of the other ſides is giuen.

As in the plaine obliquangled triangle, A B C. the angles being giuen, at A. 20. deg. 10'. at C. 60. deg. and at B. 99. deg. 27'. Likewiſe, by the 3. conſectary of the 49. of the firſt, the reaſon of the ſides is giuen after this manner,

A B. 8693512. the ſine of the angle, A C B. 60. deg. 23'.
B C. 3447521. the ſine of the angle, B A C. 20. deg. 10'.
A C. 9864292. the ſine of the angle, A B C. 99. d. 27'. or the ſine of the complement, to a ſemicircle being 80. deg. 33'.

But

But if then, beſides one of the ſides be giuen, (as for example) the ſide A B. 34. foote, the other ſides ſhall alſo be giuen, viz. B C. and A C. For.

1 As A B. 8693512. is to B C. 3447512. So is A B. 34 foote to B C. 13. $\frac{4\,0\,0\,0\,0\,0\,0\,0\,0}{8\,6\,9\,3\,5\,1\,2}$ foote. And in like manner.

2 As A B. 8693512. is to A C. 9864292. So is *A B.* 34 foote to A C. 38. $\frac{1\,0\,1\,2\,4\,7\,2}{8\,6\,9\,3\,5\,1\,2}$ foote.

Or by changing the middlemoſt tearmes.

1 As *A C B.* 8693512. is to *A B.* 34. foote. So is *B A C.* 3447512. to *B C.* 13. $\frac{12}{17}$ foote.

2 As *A C B.* 8693512. is to *A B.* 34. foote: So is *A B C.* 9864292. to A C. 38. $\frac{5}{17}$ foote.

The ſecond Conſectarie.

Two ſides being giuen, with an angle oppoſite to the one of them, the angle alſo oppoſite to the other of them is giuen.

As in the aforeſaid obliquangled triangle, *A B C.* the 2. ſides *A B.* 34. foote and *B C.* 13. $\frac{4\,0\,0\,0\,0\,0\,0\,0\,0}{8\,6\,9\,3\,5\,1\,2}$ foote being giuen, with the angle, *A C B.* 60. deg. 23', oppoſite to one of the giuen ſides, viz. to the ſide *A B*; the angle *B A C.* oppoſite to the other of the giuen ſides, to wit, to the ſide, *B C.* ſhall alſo be giuen.

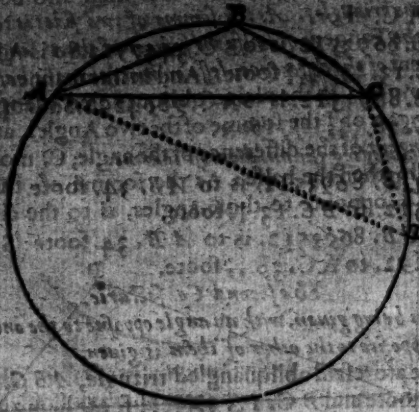
For by the angle giuen, *A C B.* 60. deg. 23. the ſine of that angle, *A B,* 5693512. is giuen.

Then I ſay: As the ſide *A B.* 34. foote, is to the ſide *B C.* 13. $\frac{4\,0\,0\,0\,0\,0\,0\,0\,0}{8\,6\,9\,3\,5\,1\,2}$ foote. So is *A B.* the ſine of the angle, *A C B.* 8693512. to *B C.* 3447521. the ſine of the angle, *B A C.*

Or the middle tearmes being changed.

As *A B.* 34. foote to *A C B.* 8693512. So is *B C.* 13. $\frac{4\,0\,0\,0\,0\,0\,0\,0\,0}{8\,6\,9\,3\,5\,1\,2}$ foote, to *B A C.* 3447521. To which ſine in the left margent of the table, the angle of 20. deg. 10. anſwereth. Therefore the angle *B A C.* is 20. deg. 10.

Note. In the vſe of this conſectarie, there may a doubt happen: that is to ſay, If 2. ſides be giuen, whereof one of them is the greateſt ſide, together with the angle oppoſite to the leſſer of the 2. giuen ſides: And the angle oppoſite to the greater of the giuen ſides be demaunded; For becauſe that angle may be either acute or obtuſe: and the ſine to both of them, is the ſame by the 1. Con: of the 12. Pro: of the 2. The doubt is, when you haue found the ſine of the angle demanded, whether that ſine ſheweth an acute, or an obtuſe angle.

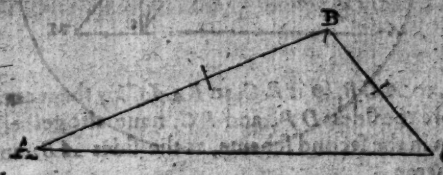


As in the obliquangled triangle, *A B C*. If the 2. sides A C. 22. foote, and *B C*. 10. foote, together with the angle *B A C*. 24. deg. 50'. 10". be giuen. And the angle *A B C*. opposite to the greatest side, A C. be demaunded, If I shall say; As the side B C. 10. foote to the side, A C. 22. foote; so is *B C*. 4200241. the sine of the angle *B A C*. to A C. the sine of the angle *A B C*. I shall readily finde 9240530. to be the sine A C. But because that sine is the sine both of the acute angle, A D C. 67. deg. 31'. 34". which the arch, *A B C*. is opposite vnto; and of the obtuse angle A B C. 112. deg. 28'. 26". which the arch A D C. is opposite vnto: It is doutfull whether the angle sheweth by that sine be the obtuse angle, 112. deg. 28'. 26". or the acute angle 67. deg. 31'. 34". Nor can this doubt be otherwise taken away, but that besides the other 3. things giuen, (which may be aswell in the acute angled triangle, *A D C*. as in the obtuse angled triangle *A B C*. For if the sides *B C*. and D C. and also the angles *B A C*. and *D A C*. are equall) this also be giuen, whether the angle sought for, be obtuse or acute: Or the same may be perceaued by the true deliniation of the triangle, to be resolued whether the angle sought for be acute or obuse.

The third Axiom.

In all plaine triangles, As the ſomme of two ſides is to their difference; So is the Tangent *of halfe the ſomme of the 2. angles oppoſite, to the* Tangent *of the difference, leſſe or more then the halfe.*

The declaration: In the plaine obliquangled triangle, *A B C.* I ſay the *Tangent* of $\frac{1}{2}$ the ſomme of the two Angles at A. and C. is to the *Tangent* of the difference of the angle, C. more, and the angle *A.* leſſe then the halfe: As the ſomme of the 2. ſides *B C.* and *A B*, oppoſite to thoſe angles, is to the difference of thoſe ſides.



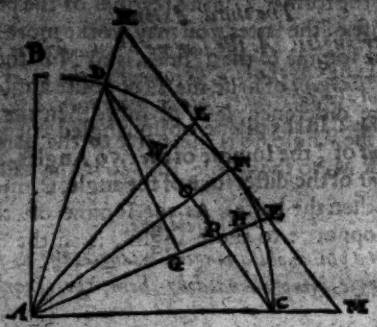
The demonſtration. For the Quadrant *A B C.* being deſcribed make the angles *D A E* and *E A C.* equall to the angles *A C B.* and *B A C.* in the former ſcheme.

And thereupon the angle *D A C.* is the ſomme of thoſe 2. angles, the halfe of that ſomme is *D A F.* or *F A C.*

The difference of the angle, *D A E.* aboue the halfe D A F. or of the the angle E A C. leſſe then the halfe, *F A C.* is the angle *F A E*; Let the ſubtenſe of the ſomme of the 2. angles, be the right line *D C.* Let the ſine of the greater angle *D A E.* be the right line D G. Let the ſine of the leſſer angle, *E A C.* be the right line, *C H.* Let F M. or F K. be the *Tangent* of halfe the ſomme of the 2. angles: Let F E. be the *Tangent* of the difference, leſſe or more then the halfe. Now the triangles G D P. and H C P. are equiangled by the 4. con: of the 49. of the 1. becauſe of the equall angles *D P G.* and *C P H.* by the 13. of the firſt and the right angles at G. and H. by the 3. con: of the 12. pro: of the firſt.

Therefore, in this ſecond ſcheme: As *P D.* to D G. ſo is *P C.* to *C H.* by the 46. of the 1. As in the firſt.

As



As *AB.* to *ACB.* ſo is *BC.* to *BAC.* by the ſecond Axiom. Therefore the ſides *DP.* and *PC.* haue altogether the ſame proportion in the ſecond ſcheme, as the ſides *AB.* and *BC.* in the firſt ſcheme.

Wherefore, putting the right line, *DC.* for the ſomme of the two giuen ſides, *AB.* and *BC.* you may take the parts *DP.* and *PC.* for the ſame 2. ſides, *AB.* and *BC.* By which poſition *NP.* is the difference of the 2. ſides. But there the ſides *AB.* and *BC.* here *DP.* and *PC.* are giuen; Therefore the difference alſo of their ſides, *NP.* and the halfe thereof, *OP.* is giuen. Moreouer, for that the compoſed Triangles, *ARLFEM.* and *ADNOPC.* are euery where equiangled, becauſe of the parallels, *DC.* and KM. Therefore the ſides and the ſegments of their ſides, are proportionall by the 46. and 47. of the firſt. And thereupon.

As DC. the ſomme of the 2. ſides, is to PN. their difference, ſo is KM. the double *Tangent* of halfe the ſomme of the 2. angles, to LE. the double *Tangent* of the difference, leſſe or more then the halfe. Or.

As OC. halfe the ſomme of the 2. ſides, is to OP. halfe their difference, ſo is FM. the *Tangent* of halfe the ſomme of the 2. oppoſite angles, to FE. the *Tangent* of the difference, leſſe or more then the halfe. Or.

Retayning

Retayning the former 2. intire tearmes of the proportion and taking the halfe of the latter, you may worke more briefliе.

As D C. the ſomme of the 2. ſides, is to N P. their difference, ſo is F M. the *Tangent* of halfe the 2. oppoſite angles, to F E. the *Tangent* of the difference, leſſe or more then the halfe. For as the whole to the whole, ſo is the parte to the parte. Therefore as the whole K M. to the whole L E. ſo is the halfe F M. to the halfe F E.

Confectarie.

Therefore in a plaine obliquangled triangle, the 2. ſides, being giuen with the angle comprehended by them, the other 2. angles are alſo giuen.



As in the plaine obliquangled triangle A B C. the two ſides A B. 6. and B C. 3. foote, with the angle A B C. being giuen 107. deg. 30′. The angles B A C. and B C A. ſhall likewiſe be giuen after this manner.

The ſomme of the 2. ſides giuen is 9. their difference is 3.
The ſomme of the angles at A. and C. is 72. deg. 30′. by the 49. of the 1.
The ½ thereof is 36. deg. 15′. whoſe *Tangent* is 7332303.
Then I ſay.
As the ſomme of the ſides giuen 9. is to their difference 3. So is 7332303. the *Tangent* of halfe the ſomme of the oppoſite angles; to 2444101. the *Tangent* of the arch of 13. deg. 44′. 4″. being the difference of the angle A. leſſe, and of C. more, then the halfe. Therefore.

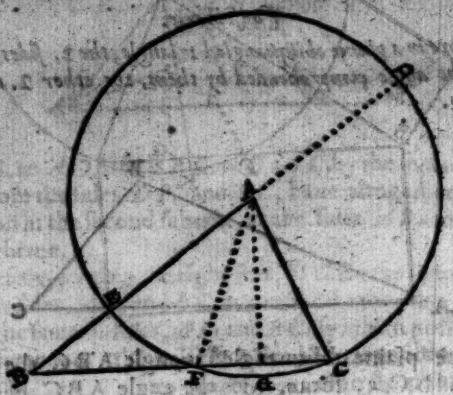
From	36. deg.	15′.	00″.	To	36. deg.	15′.	00″.
Subſtr.	13.	44.	4.	Ad.	13.	44.	04.

R. ſ angle B A C. 22. 30. 56. ſ whole is ſ angle, B C A. 49. d. 59′. 4″.

The

The fourth Axiome.

In all plaine triangles. As the greateſt ſide is to the ſomme of the other ſides, ſo is the difference of thoſe other ſides, to the ſegment of the greateſt ſide: which ſegment ſubſtracted, a perpendiculer ſhall fall in halfe the remainder.



The declaration. Let *A B C.* be an obliquangled triangle, and let the leſt ſide thereof, be A C. the greateſt *B C.* At the diſtance of the leſt ſide, A C.

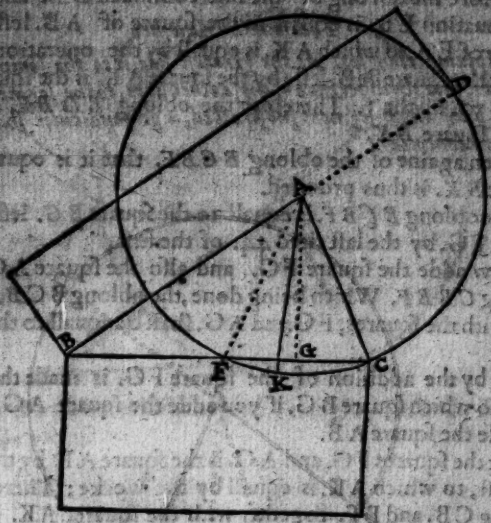
A. being the enter, let the circle C D E F. be deſcribed, cutting thother 2. ſides in the points, E. and F. Moreouer, let the ſide A B. be produced vnto D. and D B. ſhall be the ſomme of the ſides, A B. and A C. For A C. and A D. are equall by the worke, then B E. ſhall be the difference of the ſides, A B. and B C. for A E. and A C. are equall likewiſe by the worke.

I ſay firſt, that.

As C B. to B D. ſo is E B. to B F. Then that the perpendiculer A G. doth biſect the right line, F C.

The demonſtration. For as to the firſt, Equall right angled figures haue their ſides, reciprocally proportionall by the 2. Con: of the 42. of the firſt.

But



But the oblonges made of B D. and B E. and alſo of B C. and B F. are equall right angled figures.

Therefore they haue their ſides, proportionall reciprocally.

So that as B C. to B D.

So is B E. to B F.

The Minor is prooued. For whatſoeuer is equall to one and the ſame thing, are equall to one another. But the oblongs made of B C. and B F. and alſo of B D. and B E. are equall to the ſame ſquare made of the right line, B K. being the *Tangent* of the angle, B A K. Therefore alſo they are equall one to another.

Againe, the Minor is prooued. And firſt of the oblong B D. and B E. that it is equall to the ſquare, B K. is thus prooued. If a right line biſected be continued, the oblong made of the line continued, and the continuation is equall to the ſquare, made of the right line compoſed of the biſegment, and the continuation, leſſe by the ſquare of the biſegment, by the 44. of the 1. But E D. is a right line biſected in A. and continued from E. to B.

Therefore

Therefore the oblong of the line continued D B. and of the continuation E B. is equall to the ſquare of A B. leſſe by the ſquare of E A. to which A K. is equall by the operation.

But the ſquare A B. leſſe by the ſquare A K. is the ſquare *B K.* by the 50. of the 1. Therefore the oblong *B D B E.* is equall to the ſquare *B K.*

Then againe of the oblong *B C B F.* that it is equall to the ſquare *B K.* is thus prooued.

The oblong *B C B F.* is equall to the ſquare *B G.* leſſe by the ſquare F G. by the laſt cited 44. of the firſt.

Now adde the ſquare F G. and alſo the ſquare A G. to the oblong *C B B F.* Which being done, the oblong B C *B F.* together with the ſquares, F G. and A G. ſhall be equall to the ſquare A B.

For by the addition of the ſquare F G. is made the ſquare B G. to which ſquare B G. if you adde the ſquare A G. thereof is made the ſquare A B.

But the ſquares F G. and A G. is the ſquare A F. by the 50. of the firſt, to which A K. is equall by the worke; Therefore the oblong C B. and B F. together with the ſquare, A K. is equall to the ſquare, A B. And thereupon without the ſquare, *A K.* is equall to the ſquare, *A B.* leſſe by the ſquare, *A K.* that is to the ſquare B K. by the 50. of the firſt.

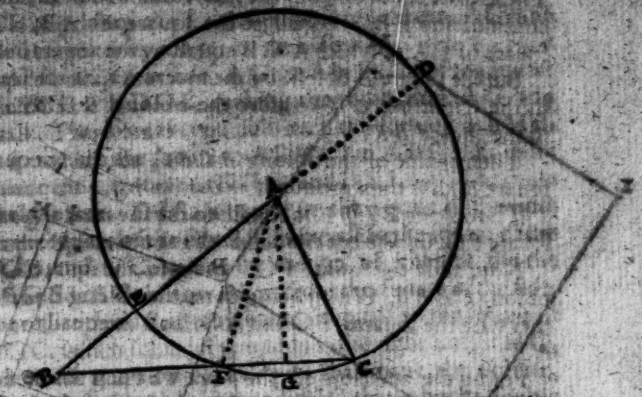
But that which I propounded in the ſecond place, of the perpendiculer, A G. biſecting the right line C F. it is thus prooued.

Becauſe the Triangle F A C. is of 2. equall ſides, in F A. and A C. by the worke. Therefore, the perpendiculer *A G.* beſecteth the baſe F G. by the 23. of the firſt.

Therefore in an obliquangled Triangle; As the greateſt ſide to the ſomme of the other ſides, ſo is the difference of thoſe other ſides, to the ſegment of the greateſt ſide, which taken away, the perpendiculer, ſhall fall in $\frac{1}{2}$ the Remainder, which was to be demonſtrated.

Conſectarie.

Therefore the three ſides of a plaine obliquangled Triangle, being giuen, whether ſegment you will of the greateſt ſide is giuen, in which from the greateſt angle, the perpendiculer ſhall fall.



As in the plaine obliquangled Triangle, *ABC.* Let the 3. sides be giuen.

AB. 20. Foote.
BC. 21. Foote.
AC. 13. Foote.

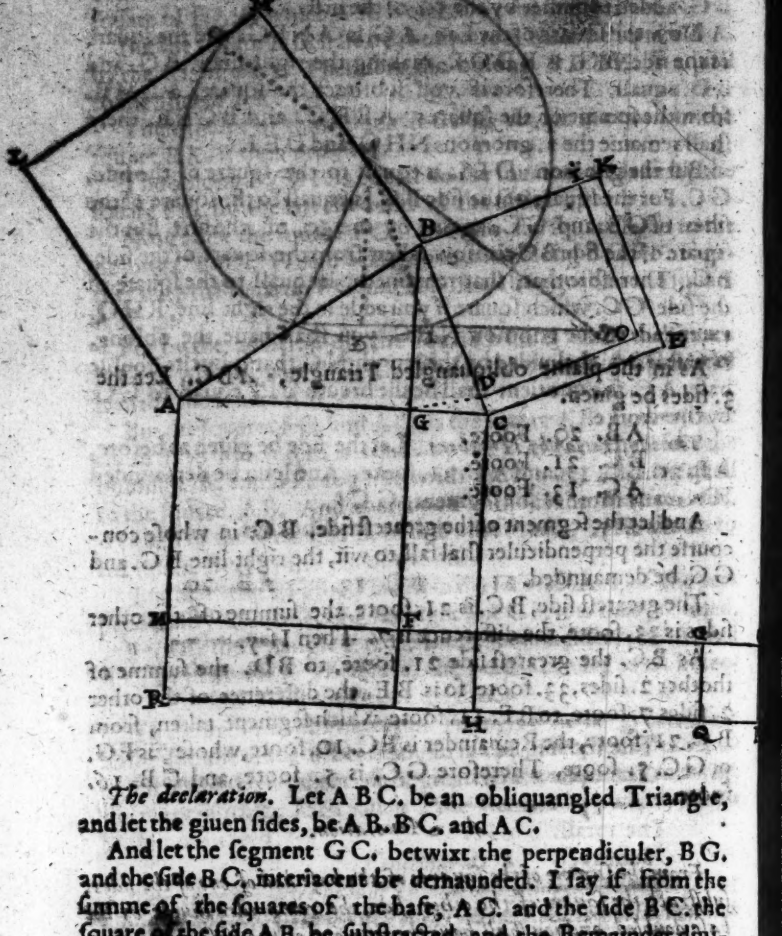
And let the segment of the greatest side, BC. in whose concourse the perpendiculer shal fall, to wit, the right line, BG. and GC. be demaunded.

The greatest side, BC. is 21. foote, the summe of the other sides is 33. foote, the difference is 7. Then I say.

As BC. the greatest side 21. foote, to BD. the summe of thother 2. sides, 33. foote, so is BE. the difference of the other 2. sides 7. foote, to BF. 11. foote which segment taken, from BC. 21. foote, the Remainder is FC. 10. foote, whose $\frac{1}{2}$ is FG. or GC. 5. foote. Therefore GC. is 5. foote, and GB. 16. foote.

An Annotation.

From the summe of the squares of the base and of one of the sides, substract the square of the other side, the remainder diuide by the base doubled, and you shall haue the segment of the base interiacent, or lying betwixt the perpendiculer, and the side first taken.



The declaration. Let A B C. be an obliquangled Triangle, and let the giuen sides, be A B. B C. and A C.

And let the segment G C. betwixt the perpendiculer, B G. and the side B C. interiacent be demaunded. I say if from the summe of the squares of the base, A C. and the side B C. the square of the side A B. be substracted, and the Remainder diuided by the base A C. doubled, the quotient shall be the segment G C.

The demonſtration. For the ſquare of the ſide A B. to wit, the ſquare A L M B. is equall to the ſquares of the ſides, A G. and B G. added together by the 50. of the firſt.

Now the ſquare of the ſide, A G. is A N F G. and the ſquare of the ſide B G. is B D O I. making the right lines, B G. and B D. equall. Therefore if you ſubſtract the ſquare, A I M B. from the ſumme of the ſquares A R H C. and B C E K. there ſhall remaine the 2. gnomons N H G. and D E I.

But the gnomon D E I. is equall to the ſquare of the ſide, G C. For the ſquare of the ſide B C. is equall to the ſquare of the ſides of G B. and G C. againe by the 50. of the firſt. But the ſquare of the ſide B G. is now taken from the ſquare of the ſide, B C. Therefore that, that remaineth, is equall to the ſquare of the ſide G C. which ſquare if you adde in the right line, R H Q. extended to the gnomon N H C. you ſhall haue the oblong, N C P R. which diuided by the length, N C. that is by the double baſe, A C. the quotient ſhall be the bredth C P. equall to C G. by the worke.

The illuſtration by Numbers. Let the ſide be giuen as before, A B. 20. B C. 13. and A C. 21. foote. And let it be demaunded how many foote is the ſegment, G C?

Anſwer 5. The whole worke ſhall be thus.

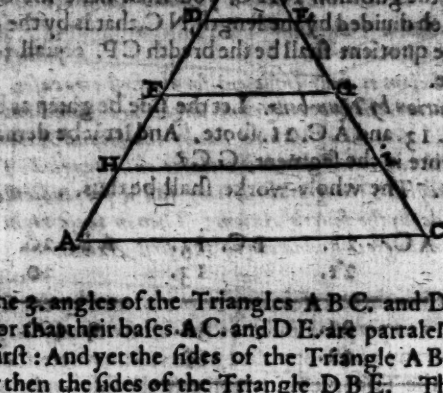
A C.	21.	B C. 13.	A B. 20.
	21.	13.	20.
	21.	39.	q. 400.
	42.	13.	
The ſquare	441.	q. 169.	
The ſquare	169.		
The totall.	610.		
Subſtract.	400.		

The Remainder. 210. Which diuided by 42. the double of A C. the quotient is 5. for G C.

The vse of the precedent Axiomes, Or

A Manuduction, wherein is shewed, how by the helpe of a few of those Axiomes, euery plaine Triangle may be resolued.

In euery Triangle there are 6. termes, to wit, 3. sides, and 3. angles: of these whatsoeuer 3. being giuen in a plaine Triangle, the other 3. may be found out, by the 4. Axiomes seruing, and sometimes diuers waies, only one case excepted, that is, if the 3. angles be onely giuen; for thereby no side can be found out. Because the 3. angles of one Triangle may be equall to the 3. angles of another Triangle, although their sides be altogether vnequall.



As the 3. angles of the Triangles A B C. and D B E. are equall, for that their bases A C. and D E. are parralell by the 38. of the first: And yet the sides of the Triangle A B C. are farre greater then the sides of the Triangle D B E. Therefore this case only is excepted in Trigonometria: In all other by whatsoeuer three termes giuen, euery fourth may be found out, which by laying downe all the cases, I will demonstrate after this manner.

A plaine Triangle is right angled or oblique-angled.

In a right angled plaine Triangle, either all the angles (that is, one of the acute angles being giuen) with one side are giuen, and the other two sides are demaunded.

2 Or

2 *Or elſe two ſides with one angle, that is; the right are giuen: and the other two angles with the third ſide, are demaunded.*

In both which caſes, the firſt Axiom is ſufficient.

In a plaine oblique angled Triangle.

1 *Either all the angles are giuen* (For, as often as two are giuen, the third is alwaies the complemẽt of the other 2. to 2. right angles by the third Conſt. of the 49. of the 1.) *with one ſide, and the other two ſides are demaunded.*

2 *Or two ſides with one angle oppoſite to the one of the giuen ſides are giuen: And the angle oppoſite to the other of the giuen ſides, together with the third ſide, is demaunded.*

3 *Or two ſides with an angle comprehended by them are giuen: And the other two angles with the third ſide are demaunded.*

4 *Or laſtly, all the three ſides are giuen: And the angles are demaunded.*

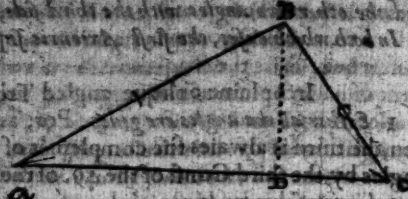
The firſt Axiom is fully ſufficient for the firſt two caſes.

In the third caſe, the two vnknowne angles are found out by the third Axiom: And then the third ſide by the ſecond Axiom.

In the fourth caſe, firſt diuide the plaine obliquangled Triangle into two right angled Triangles, by letting fall a perpendiculer vpon the greateſt ſide, by the fourth Axiom; Then in theſe right angled Triangles, euery angle is found out by the firſt Axiom.

But the former three caſes may be performed by the firſt Axiom only. So as a perpendiculer be let fall from any angle vnknowne, vpon any of the oppoſite ſides vnknowne, either within or without the Triangle (in which caſe the ſides vnknowne is to be increaſed ſometimes) and ſo the plaine oblique angled triangle will be deuided into two right angled Triangles, whether ỹ perpendiculer fall within or without the Triangle. And yet this rule (that the ſide vpon which the perpendiculer is to fall, ought to be vnknown) is only meant in the examples of the ſecond Axiom, and not in the examples of the third Axiom of plaine Triangles.

1 As if such a proportion be giuen.



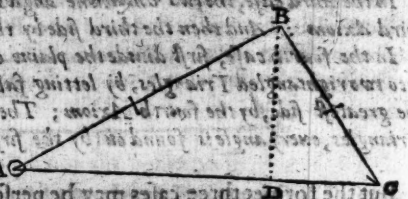
As A B. the sine of the angle A C B. to B C. the sine of the angle B A C. So is the side A B. to the side B C. by the second Axiom.

With the same effect you may say by the first Axiom.

1 As A B. the Radius, to B D. the sine of the angle B A D. So is the side A B. to the side B D.

2 As B D. the Radius to B C. the secant of the angle D B C. (which is the complement of the angle B C D. So is the side B D to the side B C.

2 If such a proportion be giuen,



As the side A B. to the side B C. So is A B. the sine of the angle A C B. to B C. the sine of the angle B A C. by the second Axiome.

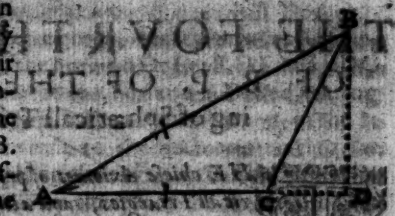
With the same effect you may say by the first Axiom.

1 As B C. the Radius to B D. the sine of the angle B C D. So is the side B C. to the side B D.

2 As the side B D. to the side A B. So is B D. the Radius to A B the Secant of the angle A B D. whose complement is the angle B A D.

And that angle A B D. added to the angle D B C. maketh the angle A B C.

3 If such a proportion were given, As the sum of 2 sides AB. and B C. to their difference: So is the Tangent of halfe the sum of the angles A B C. and A C B. to the Tangent of the difference more lesse then the halfe, by the third Axiom.



With the same effect you may say,

1 As A B. the Radius to B D. the sine of the angle B A D. So is the side A B. to the side B D.

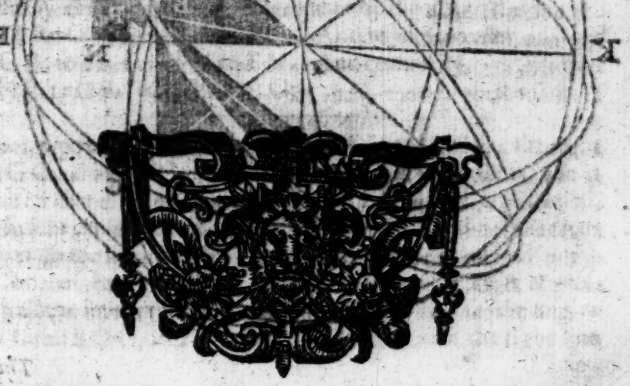
2 As A B. the Radius to A D. the sine of the angle A B D. So is the side A B. to the side A D. From whence, if you substract the side A C. there resteth the side D C.

3 As the side C D. to the side D B. So is the Radius D C. to D B. the tangent of the angle D C B. which added to the angle B A C. and the total substracted from two right angles, the remainder will be the angle A B C.

But if also you would finde the side B C. you shall likewise say by the first Axiom.

As D C. the Radius to B C. the secant of the angle D C B. So is the side D C. to the side B C.

The end of the third Booke.

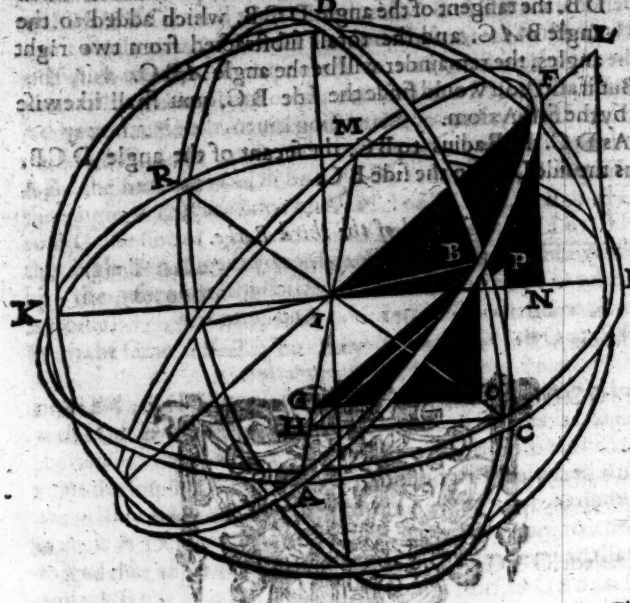


THE FOVRTH BOOKE OF B. P. OF THE MEASVRing of Sphæricall Triangles.

THE chiefe Axiomes of proportion that are in Sphæricall Triangles, and altogether sufficient for their resolution, are foure.

The first Axiome.

In many right angled Sphæricall Triangles, hauing one and the same acute angle at the base, the sines of the hipothemusses, and of the perpendiculers, are all of them proportionall one to another.



The

The Declaration.

Let K M H A D. be a Sphære giuen, and therein let K M E A be the horizon, the pole of the horizon D. Let K D E, and R D C be circles passing by the pole of the horizon K D. and cutting the horizon at right angles in K R E, and C. by the 27. of the 1. Let M E A be an oblique circle to the horizon, cutting the verticall circle K D E. at right angles at E. so that it passeth by the poles thereof M. and D. by the 17. of the 1. And in like manner is cut by it into 2. quadrants M E. and E A. by the 16. of the first. In this Sphære, and in this position of circles, there are amongst other, 2. right angled Sphericall Triangles A B C. & A E F. And in them let the hipothenusaes be A E. and A B. the perpendiculers E F. and B C. the bases A F. and A C. And the acute angles at the bases A F. and A C. let be the same angle E A F. or B A C. Lastly, let the sines of the hipothenusaes A E. and A B. be the right lines I E. and G B. and the sines of the perpendiculers E F. & B C. Let be the right lines B N. & B O. by the twelueth of the second booke. Now I say that these sines of the hipothenusaes, and of the perpendiculers, to wit, the sines I E. G B. E N. and B O. are all proportionall one to another. And so any three of them being giuen, the fourth may be found out. More plainely I say, that

As I E. to E N. so is G B. to B O. And in like manner,
As G B. to B O. so is I E. to E N. And contrarily,
As N E. to I E. so is O B. to B G.

Or by changing the middle termes by the 12. of the 1.

As I E. to G B. so is E N. to B O. And in like manner,
As G B. to I E. so is B O. to E N. And contrarily,
As N E. to O B. so is I E. to B G.

The Demonstration.

For if you ioyne together the sines G B. and B O. by the right line G O. that thereby may be made the Triangle G B O. it is manifest that the Triangles G B O. and I E N. are equiangled. For first because the right lines E N. and B O. fall perpendicularly vpon the subiected plaine M F C. by the worke, and by the 3. Con: of the 12. of the 2. Therefore they make right angles, with all the lines drawne in the same plaine, and so the angles E N I. and B O G, are right angles. Then because the right line

I E. and G B. are parallel one to another, by the 38. of the 1. for they are drawne alike vpon the same right line, I A. by the 3. Con. of the 12. of the 1. And because the whole plaine M B A. is euery where inclined with the same angle to the plaine M F A. therefore also the parallels drawen therein I E. and G B. are inclined with the same angles to the parallels I N. and G O. vnder them in the plaine M B A. and so the angles I E N. and B G O. are equall. And consequently in the Triangles I E N. and G B O. Where 2. angles are equall to 2. There also the third is equall to the third by the 49. of the 1. And thereupon the Triangles I E N. and G B O. are equiangled. But if they be equiangled, they haue the sides about the equall angles proportionall by the 46. of the 1. And so they are.

As I E. to E N. so G B. to B O. &c. which was to be demonstrated.

To this I briefly adioyne, That if the hipothenuses A E. 90. deg. and A B. 42. deg. together with the perpendiculer E F. 48. deg. 25'. be giuen: And let the perpendiculer B C. be sought for.

Of the arches giuen { A B. 90. deg. / A B. 42. / E F. 48. 25' } are the sines { I E. 10000000. / G B. 6691306 / E N. 7479912. }

Then I say, as I E. 10000000. to E N. 7479912. So is G B. 6691306. to B O. 5005038. But the arch of 30. deg. 2'. in the tables, answereth to the sine 5005038. Therefore the perpendiculer B C is 30. deg. 2'.

In like manner. Let both the hipothenuses with their sines be giuen, as before. But of the perpendiculers, let the perpendiculer B C. 30. deg. 2'. be now giuen, together with his sine B O. 5005038. And let the perpendiculer E F. be sought for. I say.

As G B. 6691306. to B O. 5005038. So is I E. 10000000. to E N. 7479912. But the arch of 48 deg. 25'. in the tables, answereth to the sine 7479912. Therefore the arch E F. is 48 deg. 25'.

Contrarily: Let both the perpendiculers E F. and B C. together with the greater hipothenusa A E. be giuen, And let the lesser hipothenusa A B. be sought for. I say. As E N. 7479912. to I E. 10000000. So is B O. 5005038. to G B. 6691306. But the arch of 42 deg. in the tables, answereth to the sine 6691306.

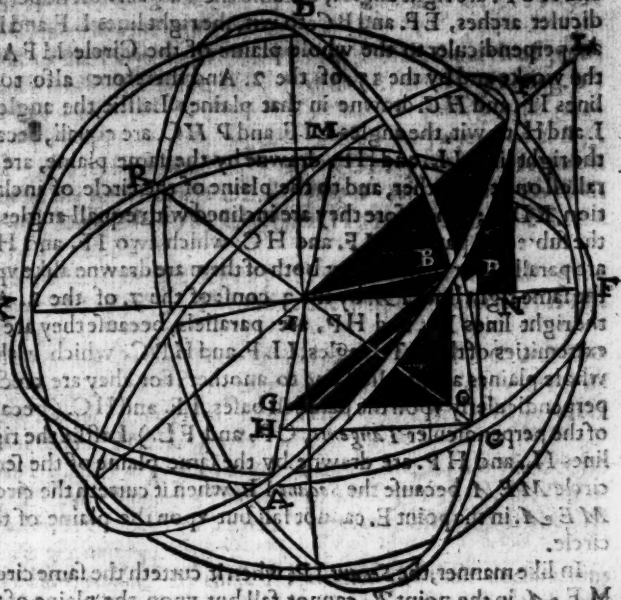
Therefore

Therefore the hipothenuse AB, is 42 deg.

The second Axiome.

In many right angled ſphæricall Triangles, having the ſame acute angle at the baſe: The ſines of the baſes and the *Tangents* of the perpendiculers are all proportionall one to another.

The declaration: In the former Diagram, and in the ſame triangles A E F. and *A B C.* wherein the ſines of the baſes *A F.* and *A C.* are I F. and *H C.* but the *Tangents* of the perpendiculers, E F. and *B C.* are L F. and *P C.* I ſay that thoſe ſines of the baſes, and the *Tangents* of the perpendiculers, that is the ſines I F. and *H C.* and the *Tangents* L F. and *P C.* are all of them proportionall one to another. And ſo, any three of them being giuen, the fourth may be found out. More plainlie I ſay that



As

As I F. to F L. So is H C. to *P C.* And in like manner,
As H C. to C P. So is I F. to F L. And contrarilie,
As F L. to F I. So is *P C.* to *C H.*

Or by changing the middle termes, by the 42. of the first.
As I F. to *H C.* So is F L. to *C P.* And in like manner,
As H C. to I F. so is *C P.* to F L. And contrarilie,
As F L. to P C. So is F I. to C H.

The demonstration. For when you haue drawne the right lines, I L. and H P. and accomplished the triangles I L F. and *H P C.* those triangles I L F. and *H P C.* shall be equiangled, and therefore proportionall in their sides, by the 46. of the first. And the triangles I L F. and H P C. shall be equiangled because of the right angles, at F. and C. and the equall angles at I. and H. and thereupon also at L. and P. by the 49. of the first. Moreouer, the angles at F. and *C.* to wit, the angles I F L. and H C P. are right angles, because the *Tangents* of the perpendiculer arches, E F. and B C. to wit, the right lines L F. and P *C.* are perpendiculer to the whole plaine of the Circle M F A. by the worke and by the 11. of the 2. And therefore also to the lines I F. and *H C.* drawne in that plaine. Lastlie, the angles at I. and H. to wit, the angles L I F. and P *H C.* are equall, because the right lines I L. and H P. drawne by the same plaine, are parallell one to another, and to the plaine of the circle of inclination, K D A. Therefore they are inclined with equall angles, to the subiected parallels I F. and H C. which two I F. and H C. are parallels for that, for that both of them are drawne alike vpon the same right line, *I A.* by the 3. conf: of the 7. of the 2. And the right lines I L. and H P. are parallels, because they are the extremities of the 2. Triangles, I L F. and H P C. which in their whole plaines are parallell one to another: for they are erected perpendiculerly vpon the parallell bases, I F. and H C. (because of the perpendiculer *Tangents*, C P. and F L.) Lastlie, the right lines I L. and H P. are drawne by the same plaine of the semicircle *M E A.* because the *Seaant* I L. when it cutteth the circle, *M E A.* in the point E. cannot fall but vpon the plaine of that circle.

In like manner, the *Seaant* I P. when it cutteth the same circle, M E *A.* in the point *B.* cannot fall but vpon the plaine of the same

same circle. Which plaine, because it is a plaine, if it should be extended according to the right line, I P. it should fall upon the *Tangent* P C. in the point P. And so the point P. should be in the plaine of the Circle, M E A. so extended. But in the same plaine is the point, H. appointed. Therefore the right line, P H. is a line falling between 2. points of the said plaine, and therupon drawne by the same plaine. All which was to be demonstrated.

The illustration by Numbers. *Let therefore the 2. bases A F. 90. deg. A C. 30. deg. 51'. 46". Together with the perpendiculer, E F. 48. deg. 25'. be given. And let the perpendiculer B C. be sought for.*

of the bases. { *A F.* 90. deg. / *A C.* 30. d. 51. 46'. } The sines { 10000000. I F. / 5129836. H C. } are. of the perpendiculer E F. 48. d. 25'. { *Tangent* is 11269872. L F.

Then I say.

As I F. 10000000. to L F. 11269872. So is H C. 5129836. to P C. the *Tangent* 5781262.

But to the *Tangent* 5781262. in the tables, the arch of 30. d. 2'. answereth. Therefore the perpendiculer B C. is 30. deg. 2'.

In like manner, let the 2. bases together with their sines be given as before: But of the perpendiculers, let now the perpendiculer B C. 30. deg. 2'. together with his Tangent *C P. 5781262. be given. And let the perpendiculer E F. be sought for.* I say: As H C. 5129838. to C P. 5781262. So is I F. 10000000. to F L. the *Tangent* 11269872.

But the arch of 48. deg. 25'. in the tables, answereth to the *Tangent*, 11269872. Therefore the perpendiculer E F. is 48. deg. 25'.

Contrarilie, let both the perpendiculers E F. and B C. and their Tangents *L F. and P C. together with the greater base, A F. and the sine thereof, I F. be given: And let the lesser base, A C. or rather the sine thereof, H C. be sought for: I say.*

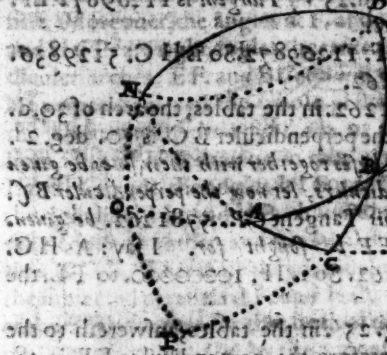
As L F. the *Tangent* 11269872. to F I. the *Radius* 10000000 So is P C. the *Tangent* 5781262. to H C. 5129838. the sine of the arch, of 30. deg. 51'. 46". Therefore, the arch or the base A C. is 30. deg. 51'. 46".

The Appendix. By these 2. Axiomes and their declarations and demonstrations, the ingenious reader may vnderstand, why there

there is no reason or proportion, betwixt the sines of the bases and the sines of the perpendiculers, and contrarilie, When notwithstanding there is proportion betwixt the sines of the hipothenuses and the sines of the perpendiculers, and contrarilie, to wit, because the sines of the bases, and of the perpendiculers doe not meete together in one same right lined Triangles. Which thing also you may sometime see that some of the most learned Mathematicians, have not noted.

The third Axiome.

In all Spheric all Triangles, the sines of their sides are duelie proportionall to the sines of their opposite angles.



The declaration. First let A B C, be a sphæricall triangle right angled at C. Then let the sides A B, A C. and C B. be continued, to make the quadrants A E, A F. and C B. and from the pole of the quadrant A F. to wit, from the point D, let be drawne downe two other quadrants, D F. and D H. And so is made three new triangles, that is the right angled triangles, B D E. and G D E. and the obliquangled triangle B D G. I say in the right angled sphæricall triangle, A B C.

As A C B. to A B. So is A B C. to A C, and so is B A C. to B C.

Or by changing the middle tearme by the 42. of the first.

As

As A C B. to A B C. ſo is A B. to A C. And
As A C B. to B A C. ſo is A B. to B C. &c. Likewiſe in the obliquangled ſphæricall triangle B D G. I ſay, that
As B D G. to B G. ſo is B G D. to B D. and ſo is D B G. to D G. &c.

The demonſtration. For as to the right angled triangle A B C. In it A C B. and A E. Alſo B A C. and E F. and on the other ſide A B C. and O P. that is the angles: and the meaſure of thoſe angles, are of the ſame quantitie.

(For as E F. is the meaſure of the angle E A F. and O P. the meaſure of the angle A B C. ſo is N D. or A E. or Q B. equall thereunto) the meaſure of the angle A C B. by the 57. of the 1.)

Therefore it is all one, if I ſhall ſay.

As A C B. to A B. So is B A C. to B C. or,
As A E. to A B. So is E F. to B C. But this is prooued by the firſt Axiom of ſphæricall triangles, and therefore that alſo.

In like ſort it is all one, as if I ſhall ſay

As *A C B.* to *A B.* So is *A B C.* to *A C.* or
As *O B.* to *A B.* So is, *O P.* to *A C.* But this is prooued by the firſt Axiom of ſphæricall Triangles, and therefore that alſo. For thoſe things that are agreeable to a third, are agreeable one to another. But by the Rules demonſtrated.

As *A C B.* to *A B.* So is *A B C.* to *A C.* And ſo is *B A C.* to *B C.* Therefore alſo

As *A B C.* to *A C.* So is *B A C.* to *B C.*

Then as to the obliquangled Triangle *B D G.* Becauſe by the demonſtration of right angled Triangles,

As *D B*, to *D E B.* So is *D E.* to *D B E.* And
As *D G.* to *D E G.* So is *D E.* to *D G E.* or (by the firſt Conſ: of the 12. of the 2.) to *D G B.* Therefore changing of the proportionall termes, Alſo
As *D G.* to *D B.* So is, *D B E.* to, or *D B G*, to *D G B.* &c.

And likewiſe, if from the point *B.* a perpendiculer be let fall to S, Becauſe then

As *B D.* to *B S D.* So is *B S.* to *B D S.* And
As *B G.* to *B S G.* So is *B S.* to *B G S.* Or by the firſt Conſ: of the 12. of the ſecond. to *B G D.* Therefore alſo
As *B G.* to *B D.* So is *B D S*, or *B D G*: to *D G B.* &c. For if

As 4 to

As 4. to 12. So is 2 to 3. And
As 2. to 12. So is 1 to 6. Then
As 2. to 4. So is 3. to 6.

The illustration by Numbers. Then in the right angled Spharicall Triangle A B C. Let the A C B. and A B C. be given, in the same quantity as before: And let the side A C. opposite to the angle A B C. be sought for. I say

As *A C B.* 90 deg. to *A B.* 42 deg. So is *A B C.* 50 deg. 3'. 12"

10000000. 6691306. 7666422

to *A C.* 30 deg. 51'. 46".

5129838

Or Contrarily. Let A B. and A C B. and A C. be given. And let A B C. be sought for, I say,

As *A B.* 42 deg. to *A C B.* 90 deg. So is *A C* 30 deg. 51'.

6691306. 10000000. 5129838.

to *A B C.* 7666422. the sine of the arch or angle of 50 d. 3'. 12"

Then let A C B. A B. and B A C. be given, And let B C. be sought for. I say,

As *A C B.* 90 deg. to *A B* 42 deg. So is *B A C.* 48 deg. 25'

10000000 6691306 7479912

to *B C.* 5005038. the sine of the arch of 30 deg. 2'.

Or Contrarily. Let A B. A C B. and B C. be given. And let B A C. be sought for: I say,

As A B. 42 deg. to A C B. 90 deg. So is B C 30 deg. 2'.

6691306. 10000000 5005038

to B A C. 7479912. the sine of 48 deg. 25'.

Lastly, Let B A C. B C. and A B C. be given: And let A C. be sought for, I say,

As B A C. 48 deg. 25'. to B C. 30 d. 2'. So is A B C. 50 d. 3'. 12"

7479912. 5005038 7666422

to A C. 5129838. the sine of 30 deg. 51'. 46".

Or

Or Contrariwise. Let B C. B A C. *and* A C. *be given. And let* A B C. *be demanded. I say.*

As B C. 30 d. 4'. to B A C. 48 deg 25'. So is A C. 30 d. 51'. 40".

5005038 7479912 5129838

to A B C. 7666422. the sine of 50 deg. 3'. 12".

In like manner, in the obliquangled Spherioall Triangle B D G. *First, let* D B G. D G. *and* B D G. *be given. And let* B G. *be sought for. I say.*

As D B G. 50 deg. 3'. 12". to D G. 45 deg. 57'. 41". So is B D G. 28 deg. 14'.

7666422 7188714 4730634

to B G. 4435860 the sine of 26 deg. 19'. 58".

Again. Let B G. B D G. *and* D G. *be given. And let* D B G. *be sought for: I say,*

As B G. 26. d. 19'. 58". to B D G. 28 14'. So is D G. 45 d. 57'. 41"

4435860 4730634 7188714

to D B G. 7666422 the sine of 50 deg. 3'. 12".

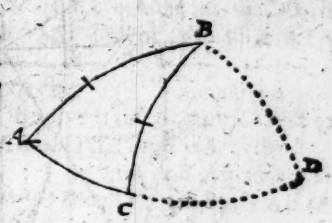
Lastly, let D G. D B G. *and* D B. *be given, And let* D G B. *be demanded. I say,*

As D G. 45. d. 57'. 41". to D B G. 50 d. 3'. 12". So is D B. 59 d. 58"

7188714 7666422 8657344

to D G B. 9232491. the sine of the obtuse angle 112 d. 35'. 40"

Note. In the vse of this Axiom the same doubt may fall out as I have formerly said, might happen in the vse of the second Axioms of plaine Triangles: As appeareth by the like Scheme A B C D. Therefore it behoueth you to be diligent, least in such case you be deceiued in finding an acute angle for an obtuse, or Contrarily.

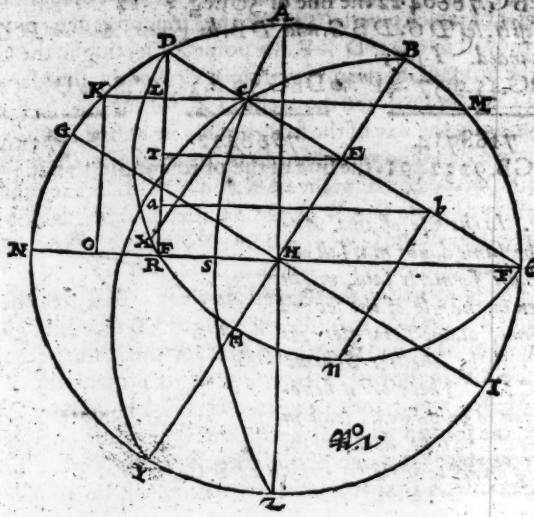


The fourth Action.

In all Sphæricall Triangles. If first you adde the two sides either of them lesse then a quadrant, together, and then adde the lesser side to the Complement of the greater side; And if you substract the sine of the Complement of the former composed arche from the sine of the latter composed arch: Or if you adde it to the sine of the excesse. Then

As the Radius is to that halfe of the right line, so made either by addition or substraction. So is the versed sine of the angle comprehended of the said two sides to a right line, which substracted from the sine of the latter composed arche, leaueth the sine of the Complement of the third side: Or from whence the sine of the latter composed arche substracted, leaueth the sine of the excesse of the third side.

Or Contrarily. As the halfe of that right line to the Radius: So is the right line made of the sine of the latter composed arche, either by substraction of the sine of the Complement of the third side, or by addition of the sine of excesse of the same third side, to the versed sine of the angle comprehended of the other two sides.



The

The Declaration. This Axiom hath diuers cases.

For first, the two sides including the angle giuen or sought for, added togeather, are either equall or vnequall to a quadrant, and that either lesse or more.

Then the angle giuen or sought is either right or oblique, and that either acute or obtuse.

Lastly, the third side opposite to the said angle, is lesse or more then a quadrant. All these cases, I shall plainely explaine as I thinke, in three schemes. In euery of which, the obliquangled Triangle, for example, is A B C. wherein either the two sides A B. and B C. are giuen togeather with the angle at B. And the third side A C. is sought for; or els all the three sides are giuen, and the angle opposite to the third side A C. is sought for.

Moreouer, of the two sides A B. and B C. including the angle giuen, or sought for (which is alwayes placed at B.) A B. is the lesser, and B C. the greater side.

The arch G N. is equall to the lesser side A B. by the worke. From the greater side B C. let the equall arches B F. and B D. be cut off by a parallel, described on the superficies of the Globe; B. being the pole, and B C. the distance of the compasses. the Diameter of which parallel is D C F. the circumference, (only noted in the first scheme) D X F. the point X meeting in the Globe with the point C. of the great circle B C. And in that same parallel D X F. let be noted the arche D X. for the measure of the angle at B. by the sixe of the first, and his right sine X C. by the 12. of the 2. and the versed sine D C. by the 13. of the second.

Lastly, let the equall arches A K. and A M. in like sort, be cutte off from the Circle D A B. by the parallel K C M. described in the superficies of the Globe, A being the pole, and the distance of the compasses A C.

These things being thus layed downe, First, let the sides A B. and B C. or B F. including the angle A B C. either giuen or sought for, be added togeather, And let the former composed arche be A F. in the first scheme, equall to a quadrant: In the second lesser, and in the third more then a quadrant. And let V F. in the second scheme (being the sine of the complement) and in the third (the sine of the excesse) be noted. Then let the lesser side A B. that is by the worke G N. be added to the

ſine of the complement of the greater ſide G D. and let D N. be the latter compoſed arch, and his right ſine D P. From which ſine D P. let the ſine of the complement V F. or P R. (in the ſecond ſcheme) be ſubſtracted: But in the third ſcheme, let the ſine of the exceſſe V F. or P R. be added to the ſine D P. that thereby the right line D R. may be found, which ioyned with the right line D F. by the right line R F. maketh the plaine rightangled Triangle D R F. by the halfe whereof let the right line T E. be drawne, cutting the right line D F. in E. by the worke, and ſo alſo the right line D R. by the 45. of the firſt. making the Triangle D T E. equiangled to the Triangle D R F. by the 38. of the which Triangle D T E. thus made, I ſay that

As the Radius E D. to the halfe of the right line D R. to wit, to the right line D T. So is the verſed ſine of the angle *A B C.* to wit the right line *D C.* to the right line *D L.* which taken from the ſine of the latter compoſed arch *D P.* there remaineth the right line L P. or K O. by the 39. of the 7. being the right ſine of the arch K N. or C S. being the complement of the third ſide *A C.*

And Contrarily.

As the halfe of the right line being *D T.* is to the Radius *D E.* So is the right line *D L.* (after the ſubſtracting of the ſine of the Complement of the third ſide from the other ſine D P.) to the verſed ſine *D C.* &c.

The Demonſtration.

For the Triangles *T D E.* and *L D C.* are equiangled by the worke, and by the 38. of the firſt. Therefore their ſides about the equall angles are proportionall by the 46. of the firſt. Nor is it any obſtacle that the right lines *D C.* and *D E.* are deuided into leſſer parts then the right lines *D L.* and *D T.* becauſe the Radius *D E.* is leſſer then the Radius G H, with which Radius G H. the right lines *D L.* and *D T.* are diuided into equall parts.

For it is no matter into how many parts ſoeuer one or another ſide of the plaine Triangle be diuided: So that the like ſide be diuided with the like into the ſame equall parts, that is,

the

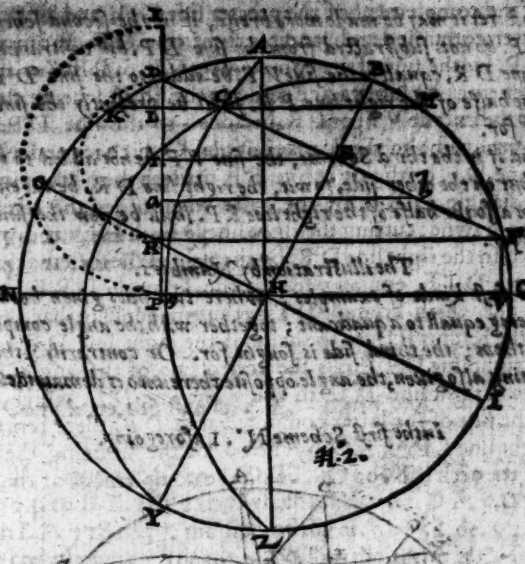


Fig. 2.

the perpendiculer with the perpendiculer, the hipothenusa with the hipothenusa, and the base with the base.

As for Example in the Triangles *A B C.* and *D B E.* It matters not whether I shall say,

As A B. 10. to *D* B. 5. So is B C. 3. to B E. 1 ½ or

As *A* B. 5 to *D* B 2 ½ So is B C. 3 to B E, 1 ½.



Contrarie. By this declaration & demonstration it appeareth: If the angle given at B. be a right angle, and his versed sine E B. the Radius; in that case, that is no needs of either multiplication or division: but by addition and substraction only, the sine of the Complement of the third side may be found out. Which briefe rule of calculating of Triangles is more precious then any golde.

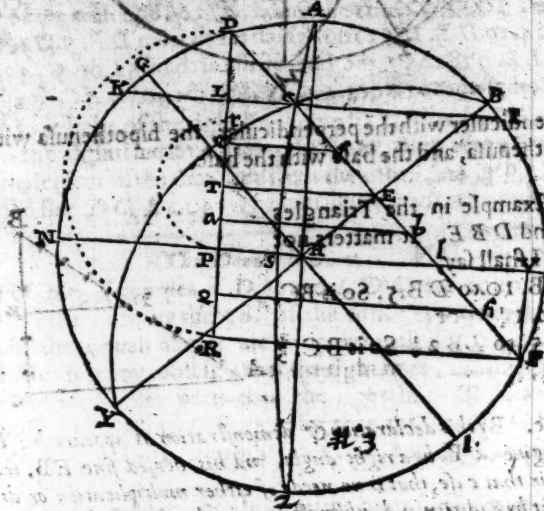
And yet it may be made more breife, if in the second scheme the fine V F. be not substracted from the fine D P. but contrariwise, the right line D R. equall to the fine V F. be added to the fine D P: For then the halfe of the right line P R. shall be presently the fine T P. sought for.

And if in the third Scheme, the fine V F. be not added to the fine D P. but on the other side, to wit, the right line D R. be taken from it, then also the halfe of the right line R P. shall be now the fine T P. sought for.

The illustration by Numbers.

The first kinde of examples: where two sides given both together, being equall to a quadrant; together with the angle comprehended by them; the third side is sought for. Or contrarily: the third side being also given, the angle opposite thereunto is demanded.

In the first Scheme N°. 1 aforegoing.



1 If the angle given, be a right angle; And his versed sine DE, AB. 35. d

AB. 35 d. 40'. the same 35 d. 40'.
BC. 54 20. the Compl. 35. 40.

AE. 90. d. DN 71. 20'. 9473966.
DT. or TR. is — 4736983. the sine of the arch of 28 deg. 16'. 29", whose complement 61 deg. 43'. 31". is the arch AC. sought for.

2 If the Angle given be acute, and his versed sine DC,
AB. 35 d. 40', the same 35 d. 40'.
BC. 54 20. the Compl. 35. 40.

AE. 90. — DN is 71. 20. DP. is 9473966
DT is 4736983

ABC. 50 deg. the Radius DE. 10000000.
the Compl. 40. the sine is, CE. 6427876.
DC. 3572124.

As EB. 10000000. to DT. 4736983. So is DC. 3572124. to DL. 1692109. which taken from DP. 9473966 leaueth LP. 7781857. the sine of the arch of 51 deg. 5'. 41". whose complement 38 deg. 54'. 19". is the arche AC. sought for.

3 If the angle giuen be obtuse, and his versed sine DB.
AB. 35 deg. 40', the same 35 deg. 40'.
BC. 54 20. the Complement 35. 40.

AE. 90. — DN. 71, 20 DP. 9473966
DT. 4736983

ABC. 112 deg. 35'.

90. — . DE. 10000000
22. 35 EB, 3840267
DB. 13840267

As DE. 10000000. to DT. 4736983. So is DB. 13840267 to DA. 6556111. which taken from DP. 9473966. there remaineth AP. 2917855. the sine of the arch of 16 deg. 57'. 53". whose complement 73 deg. 2'. 7". is the arch AC. sought for.

4 If the third ſide be giuen, which there is alwayes leſſer then a quadrant, As for example, if the ſide A C. be giuen, and the angle A B C. be ſought for.

A B. 35 d. 40′. the ſame 35 d. 40′.
B C. 54 d. 20′ the compl: 35. 40

A F. 90. — *D N.* — 71. 20. *D P.* 9473966
A C. 38 deg. 54′. 19″ — *D T.* 4736983
Com: 51. 5. 41 — — *L P.* 7781857
D L. D E, 1692109

As *D T,* 4736983, to *D E,* 10000000. So is *D L.* 1692109 to *D C.* 3572124. which taken from *D E.* 10000000. leaueth C E. 6427876. the ſine of the arch of 40 deg. whoſe Complement 50 deg. is the angle A B C. ſought for.

The ſecond ſort of examples. Where the two ſides being giuen both together leſſe then a quadrant, with the angle comprehended of them: the third ſide is required. Or Contrarily, the third ſide being alſo giuen, the angle oppoſite thereunto is demaunded.

In the ſecond Scheme N°. 2.

1 If the angle giuen be a right angle, and his right ſine *D E.*

A B. 28 d. 15′, the ſame 28 d. 15′.
B C. 40. 30. the Comp. 49. 30.

A F. 68. 45 *D N.* 77. 45. *D P.* 9772311
F Q. 21. 15 *F V.* or P R. or *D R.* — 3624280
P R 13396691 09
T P. 6698345

Being the ſine of the arch of 42 deg. 3′, 15″, whoſe complement 47 deg. 56′, 45″, is the arche A C. required.

2 If the angle giuen be acute, and his verſed ſine *D C.*

A B. 26 d. 20′, the ſame 26 d. 20′.
B C. 59 58 the Comp: 30 02

A F. 86 18 *D N.* 56 22. *D P.* 8325991
F Q. 03 42 — — *V F.* 645323
D R. 7680668
D T, 3840334

A B C

ABC. 50. deg. 4'. 10000000
32. 56. 6418958

DC. 3581042

As DE. 10000000 to DT. 3840334. So is DC. 3581042. to DL. 1375239. which substracted from DP. 8325991. leaueth LP. 6950752. the sine of the arch 44 deg. 2'. whose complement 45 deg. 58'. is the arch AC. sought for.

3 If the angle giuen be obtuse, and his versed sine DB.

AB. 26. deg. 20. the same 26. 20'.
BC. 45. 58. the com. 44. 02.
AF. 72. 18. DN. 70. 22. DP. 9418621.
FQ. 17. 42. ——— FV. 3040331.
DR. 6378290
DT. 3189145.

ABC. 112. 35.
90. DE. 10000000
22. 35. EB. 3840267.
DB. 13840267.

As DE. 10000000. to DT. 3189145. So is DB. 13840267 to DA. 4413861. Which substracted from DP. 9418621. leaueth AP. 5004760. the sine of the arch, of 30 deg. 1'. 53". Whose complement 59. deg. 58'. 7". is the arch, AC. required.

4 If the third side, be giuen: which also in these kinde of examples is alwayes lesse then a quadrant, as for example: if the side AC. be giuen, and the angle ABC. be demaunded.

AB. 26. 20. The same 26. 20'.
BC. 59. 58. The com. 30. 02.
AF. 86. 18. DN. 56. 22. DP. 8325991
FQ. 3. 42. ——— VF. 645323
DR. 7680668
AC. 45. 58. DT. 3840334
44. 02. ——— LP. 6950752
DL. 1375239

As DT. 3840334. to DE. 10000000. So is DL. 1375239. to DC. 3581042. the versed sine: which taken from DE. 10000000.

10000000. leaueth CE. 64189458. the right sine of the angle, CBE. 39. deg. 56'. whose complement 50. deg. 4' is the angle *ABC*, demaunded.

The third kinde of examples: where 2. sides giuen being together more then a quadrant, with an angle comprehended by them, the third side is demaunded: or contrariwise, the third side being also giuen, the angle opposite thereunto is required. In the 3. scheme No. 3.

1 If the angle giuen be a right angle, and his versed sine DE.

AB. 40. d. 06'. The same 40. d. 06'.
BC. 72. 12. The com. 17. 48.

AF. 112. 18. DN. 57. 54. DP. 84712119
QF. 22. 18. VE. PR. or DR. — 3794502

RP. 4676657
TP. 2338328

The sine of the arch, of 13. deg. 31'. 22". whose complement *AC*. 76. deg. 28'. 38". is the third side, demanded.

2 If the angle giuen be acute, and his versed sine *DC*.

AB. 45. d. 58'. The same 45. d. 58'.
BC. 59. 58. The com. 30. 02.

AF. 105. 56. DN. 76. — DP. 9702957
QF. 15. 56. — VF. 2745187

ABC. 28. d. 14. DE. 10000000. DR. 12448144
61. 46. CE. 8810284. DT. 6224072

DC. 1189716.

As DE. 10000000 to DT. 6224072. So is DC. 1189716 to DL. 740488. which taken from DP. 9702957. leaueth LP. 8962469. the sine of the arche 63 deg. 40'. 8". whose complement 26 deg. 19'. 52". is the side AG. sought for.

3 If the angle giuen be obtuse: and his versed sine Bb.

AB 45 deg. 58'. the same 45 deg. 58'.
BC 59. 58. the Comp. 30 02.

AF.

AF, 105, 56, DN, 76, DP, 9702937
QF 15, 56, VF, 2745187

ABC, 112 4, 35', DR, 12448144
ſ exceſſe 22, 35, DB, 15840267, DT, 6224072

As DE 10000000 to DT 6224072, ſo is Db. 15840267 to DA. 8614282, which taken from DP, 9702957, leaueth AP, 1088675, the ſine of the arch 6 deg, 15', whoſe complement 83 deg, 45', is the third ſide AC required.

4. If the angle giuen be obtuſe, and his verſed ſine DH,

AB, 45 deg. 58' the ſame 45 deg, 58',
BC, 59 58 ſ comp, 30 02

AF, 105 56 DN, 76 DP, 9702957
QF, 15 56 VF, 2745187

DR, 12448144

ABC, 170 TD, 6224072
90

ſ exceſſe 80 DH, 19848077,

As DE, 10000000 to DT, 6224072, ſo is DH, 19848077 to DG, 12353586, from whence the ſine DP, 9702957, ſubſtracted,

Leaueth PG, 2650529. The ſine of the arch, of 15, deg. 22', 14", which added to the quadrant 90, deg, maketh AC, the third ſide demanded to be 105, deg, 22', 14",

Note, If in this caſe the fourth number be found all one with the ſine DP, as it ſhould be found, if the verſed ſine were DI, it is a ſigne that the third ſide is a quadrant, becauſe it hath no ſine of the complement, or of the exceſſe. For if you ſubſtract DP, from DP, there remaineth nothing.

5. If the third ſide, be giuen leſſe then a quadrant, and his verſed ſine LP,

AB,

AB, 45. deg, 58'. The same 45. deg. 58'.
BC. 59. 58. The com. 30. 02.

AF. 105. 56. — DN. — 76. DP. 9702957
QF. 15. 56. — VF. 2745187

AC. 26. deg. 20. DR 12448144

TD. 6224072

63. 40. 63. deg. 40. — LP. 8962469

DL. 740488

As DT. 6224072. to DE. 10000000. So is DL. 740488. to DC. 1189716. Which substracted from DE. 10000000. there remaineth CE. 8810284. the sine of the angle CBE. 61. deg. 46'. whose complement 28. deg. 14'. is the angle ABC, required.

6 If the third side be giuen more then a quadrant, and the sine of his excesse PG.

AB, 45, deg, 58', The same 45, d, 58',
BC, 59, 58, The com, 30, 02,

AF, 105, 56. DN. — 76. — DB. 9702957
QF, 15 56. VF. 2745187

DR 12448144

DT 6224072

AC, 105, deg, 22',
Excesse 15, 22, PG 2650829

DP 9702957

DG 12353586

As DT. 6224072 to DE. 10000000. So is DG. 12353586 to DH. 19848077, the versed sine of the angle, ABC, demaunded being 170. deg.

The

The vſe of the aforegoing Axiomes, Or

A direction, whereby is ſhewed, how by the helpe of theſe 4. Axiomes, which formerly haue beene explained, any demaund, in whatſoeuer ſphæricall triangle, may very eaſily be found out.

Firſt Remember, that ſome ſpharicall triangle is right angled, and ſome obliquangled. And that of right angled ſpharicall triangles, ſome haue 3. ſome 2. and others only one right angle.

Therefore if a right angled ſpharicall triangle, haue 3 right angles, thoſe three right angles being giuen; their ſides alſo are giuen. And contrarilie by the 68. of the firſt.

If the right angled ſpharicall triangle haue two right angles: theſe two right angles being giuen, the two ſides alſo oppoſite to theſe 2. right angles, are giuen; to wit, 2. quadrants by the 68. of the firſt. But if beſides the third ſide be alſo giuen; or the third angle, either of theſe being giuen; th'other alſo ſhall be giuen; for that the third ſide oppoſite to the third angle, the ſides being 2. quadrants; is nothing elſe but the meaſure of that angle by the 58. of the firſt.

Therefore in theſe two caſes, there is no neede of Trigonometrie. *But if the ſpherical right angled triangle, haue onely one right and the other two obliqne angles; in that caſe* Trigonometrie *is often required.*

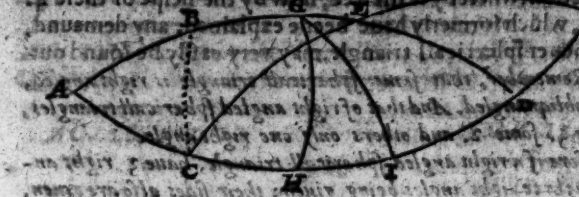
And ſithence a right angled ſpharicall Triangle, of this ſort is threefould: for either both the other two angles are acute, or both obtuſe, or one obtuſe and the other acute by the 63. of the firſt. My Axiomes ſhew not the reſolution of them, except they haue beſides the right angle two acute angles, and by that meanes euery ſide leſſe then a quadrant by the 65. of the firſt.

But if a right angled ſpherical Triangle, with two obtuſe angles be giuen you to reſolue, or with one obtuſe and one acute angle: or with two ſides either of them more then a quadrant; In ſtead of that Triangle, you may reſolue the leſſer Triangle, oppoſite thereunto. As,

Let the right angled triangle, B D C. right angled at D. and obtuſe angled at B. and C. be giuen you to reſolue, in ſteede thereof you may reſolue the right angled triangle, A B C. oppoſite from the angle *D.* to the triangle *B D C.*

For whatſoeuer 3. things are giuen in the triangle, *B D C.* the ſame 3. things ſhall be alſo giuen in the triangle, A B C. ſithence the angles at A. and *D.* are equall by the 59. of the firſt: but the

ſides



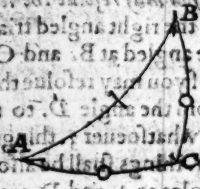
ſides A B. and B C. are the complements of the ſides, *B D.* and *C D.* And ſo the the obtuſe angles at B. and C. are the complements of the acute angles, at B. and C. by 60. and 11. of the firſt.

In like manner, If the triangle C E D right angled at D, obtuſe angled at E, and acute angled at C. be giuen you to reſolue, or finde thereof you may reſolue the triangle E D F. oppoſite to the triangle E C D. from the angle C.

But if a right angled ſphericall Triangle, with two acute angles, or with all the 3. ſides euery one of them being leſſe then a quadrant, be giuen you to reſolue: therein nothing can be demaunded that you may not finde, by the helpe of a fewe of my Axiomes, one of whatſoeuer three thinges giuen, either with one multiplication, or diuiſion, and ſometimes alſo without any multiplication or diuiſion, by addition and ſubſtraction onely: Prouided alwayes that if in the triangle propounded, a ſufficient proportion for the reſolution be not apparent betwixt the thinges giuen, and the things demaunded, you may then continue euery of the ſides vntill quadrants. And ſo conclude the whole figure in a quadrant.

This being done, in the complements of the ſides, and angles giuen and required, you ſhall finde ſome proportion, fitting to your purpoſe.

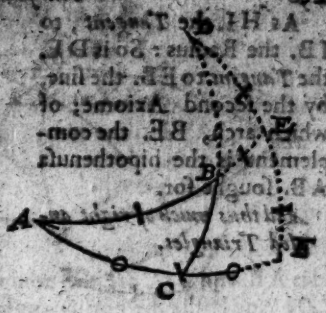
As for example. In the triangle ABC By the ſide A B. and the angles B A C, and A C B. the ſide A C. is required: Becauſe there is no proportion, ſhewed in theſe things giuen and demanded, whereof mention is made in the Axiomes of proportions, therefore you may continue euery of the ſides, vnto quadrants, and conclude the whole fi-



gure

gure in the quadrant, *D F.* after this manner; which continuation being made in the *B D E.* and *C D F.* ſuch a proportion is giuen, as it as ſet downe in the ſecond Axiome.

Therefore by that Axiome you ſhall thus conclude.

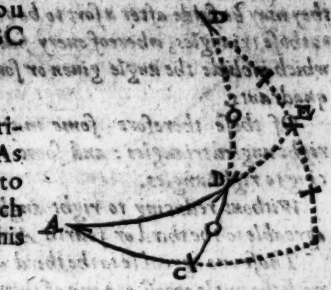


As the ſine of the baſe *D E.* to the *Tangent* of the perpendiculer *E B.* So is the ſine of the quadrant *D F.* or the *Radius* to the *Tangent* of the perpendiculer *F D C.* whoſe complement is the arch, *A C.* required.

Likewiſe, If in the Triangle A B C. all the angles are giuen, and the perpendiculer B C. is required, becauſe in theſe things giuen and ſought for, there is no proportion manifeſt, according to my Axiomes, therefore you may continue the triangle ABC after this manner.



Which being done, in the triangle, D E B. it ſhall be. As D B E. to D E. So is D E B. to D B. by the third Axiome, which D B. being knowne, B C. his complement is alſo knowne.

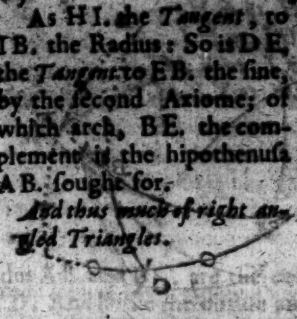


But if the firſt continuation be not ſufficient, you may alſo make the ſecond, As you ſee done in this example; Where by the 3. angles giuen, to finde the hipothenuſa, the firſt continuation is not ſufficient, therefore I haue made the ſecond, that is, I haue alſo continued the triangle B D E. as formerly I had continued the triangle, A B C. which being done, the proportion is.

As

As H I. the *Tangent*, to I B. the Radius: So is D E. the *Tangent* to E B. the sine, by the second Axiome; of which arch, B E. the complement is the hipothenusa A B. sought for.

And thus much of right angled Triangles.



Of oblique angled Triangles.

Touching obliquangled triangles, you are for the most part to be aduertised as in right angled: to wit, if an obliquangled triangle be giuen you to resolue, hauing the sides euery one more then quadrants, in stead thereof you may resolue the triangle opposite thereunto, that hath the sides euery one lesse then quadrants: which opposition you haue learned in the first booke, the 60. Pro: For my Axiomes of proportions, albeit they may be set forth after a sort to be generall, yet they are chiefly applied to those triangles, whereof euery side, or two of the principall (that is which include the angle giuen or sought) are euery of them lesse then quadrants.

Of these therefore some may be resolued without reduction to right angled triangles: and some cannot be resolued without reducing to right angles.

Without reducing to right angles, they may be resolued which are agreable to the third or fourth Axiomes of proportions.

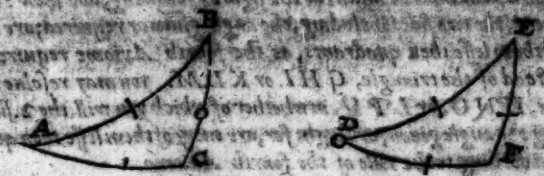
Those are agreable to the third Axiome, wherein: by the 2. sides with the angle opposite to one of them, the angle opposite to the other of them is required: Or contrarily by the two angles, with one side opposite to one of them; the other side opposite to th'other of them is demanded. As in these Diagrams following. Wherein.

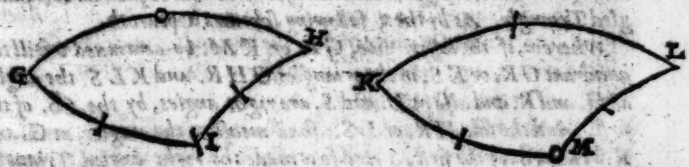
As GIH. to GH.
So is HGI. to HI.
And as KL. to KML.
So is KM. to KLM.

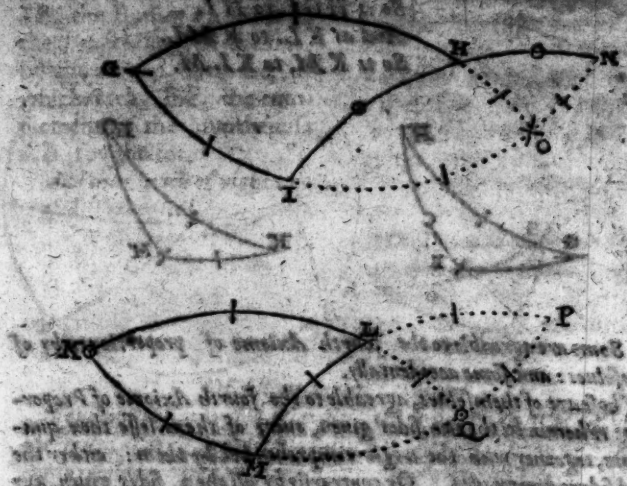


Some are agreable to the fourth Axieme of proportions, by of themselues: and some accidentally.

Those are of themselues, agreable to the fourth Axieme of Proportions: wherein by the two sides giuen, euery of them lesse then quadrants, together with the angle comprehended by them: either the third side is demanded. Or contrarily by all the 3. sides giuen, any angle, comprehended of two of the sides, euery of them being lesse then quadrants, is required: As in these.







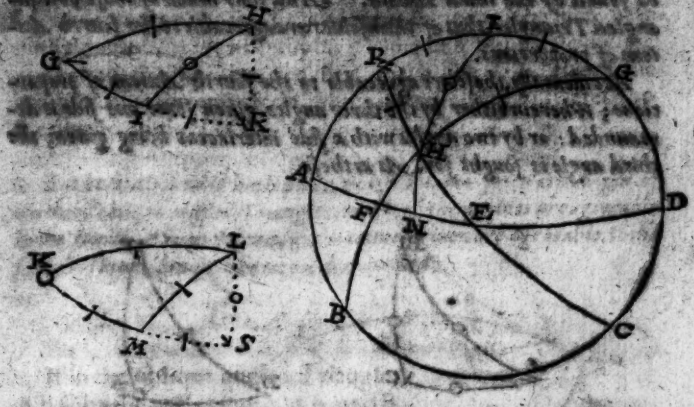
In the 2. latter whereof, which haue the ſides, G H. and K L. greater then Quadrants.

If ſuch a demaund be made, as the figures following, doe ſhew: becauſe there, the two ſides including, the angle giuen or required, are not euery of them leſſe then quadrants, as the fourth Axiome requireth.

In ſtead of the triangle, G H I. or K L M. you may reſolue the Triangle, H N O. or L P Q. in whether of which you will, the 2. ſides including the angle giuen, or ſought for, are euery of them leſſe then quadrants, according to the rule of the fourth Axiome.

But if the ſide G H. or K L. be a quadrant, you neede not reſolue the obliquangled Triangle, G H I. or K L M. but you may reſolue the right angled triangle, which alwayes lyeth next to ſuch an obliquangled Triangle, As by the 3 following ſchemes appeareth.

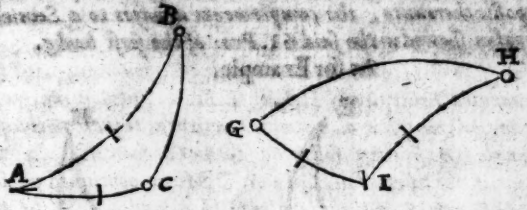
Wherein, if the leſſer ſide, G I. or K M. be continued vntill the quadrant G R. or K S. in the triangles G H R. and K L S. the angles at H. and R. and alſo at L. and S. are right angles, by the 68. of the firſt. And the ſide H R. or L S. ſhall meaſure the angles, at G. and K. by the 58. of the firſt. And ſo is made the right angled Triangle, I H R. or M L S. of 3. giuen tearmes, which right angled Triangle, being



being resolued; the obliquangled Triangle, adiacent thereunto (for that it containeth the complements of the right angled triangle) shall be resolued.

These thinges being obserued: the fourth Axiome shall be sufficient: nor shall I neede, for euery case of obliquangled Triangles, to make a particuler Axiome, which otherwise should be done.

But this also in this place you are to obserue: If the termes giuen, of an obliquangled Triangle propounded, be agreeable to the fourth Axiom; and yet not the termes demaunded, As in these:



In one of which, the angle B and C. and in the other the angle at G and H. as demaunded: First the side B C. and G H. is to be found by the fourth Axiom; then by that found, any of the other angles may

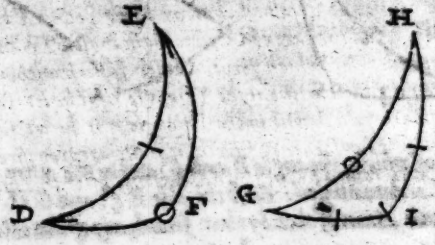
be found by the third Axiom. And thus much of those obliqueangled Triangles, that of themselues are agreeable to the fourth Axiom of proportions.

Accidentally those are agreeable to the fourth Axiom of proportions; Wherein either by the three angles giuen some one side is demaunded: or by two angles with a side interiacent being giuen, the third angle is sought for. As in these.



Which I therefore say accidentally, to agree to the fourth Axiom, because otherwise they are not agreeable thereunto, then that the sides may be changed into angles, and the angles into sides; which how the same may be performed, I haue shewed in the 1 booke, the 61. Propo: which proposition, he that truly vnderstandeth, and well weyeth with himselfe, shall here neede no more of that matter. Yet in fauour of the learners which do not alwayes retaine the speciall points of rules set downe, I will here insert, and repeat the same: in this changing of angles and sides, you must take in steed of the greatest side, and the angle opposite thereunto, the complements alwayes to a Semicircle; for the reason shewed in the said 61. Pro: of the first booke.

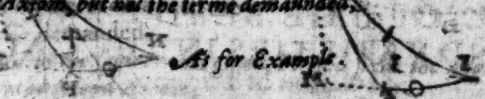
As for Example.



If in the triangle D E F, you would change the angles into sides reciprocall contrarily, the Triangle will thereupon be such, as is the Triangle G H I. whereupon it appeareth in the calculation, you are not to take the verſed ſine of the ſide D E. but of the complement to a ſemicircle, which complement anſwereth to the obtuſe angle H I G.

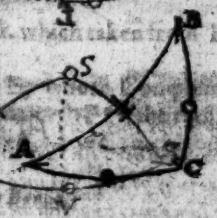


But here alſo, that hath place which I have forewarned you of touching obliqne angled Triangles, which of them ſelves are agreeable to the fourth Axiome; to wit, if the terme given are agreeable to the fourth Axiom, but not the terme demaunded.



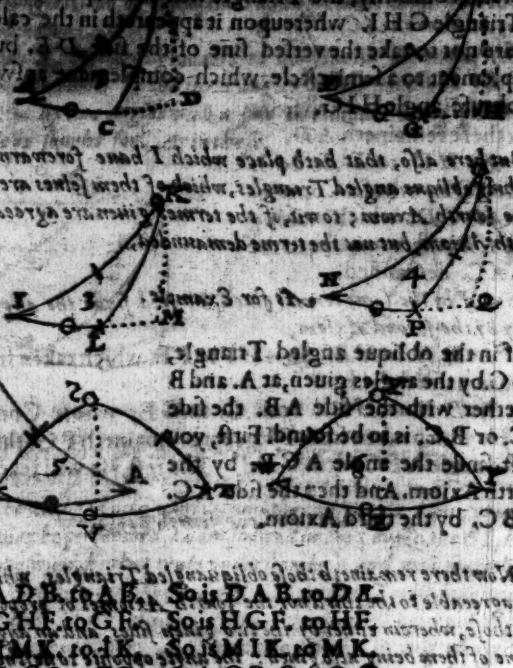
As for Example.

If in the oblique angled Triangle, A B C. by the angles giuen, at A. and B together with the ſide A B. the ſide A C. or B C. is to be found. Firſt, you muſt finde the angle A C B. by the fourth Axiom. And then the ſide A C. or B C. by the third Axiom.



Now there remaineth thoſe obliquangled Triangles, which neither are agreeable to the third nor the fourth Axiome of proportions: to wit thoſe, wherein neither by the two giuen ſides, and an angle oppoſite to one of them being alſo giuen: the angle oppoſite to neither of them, or the ſide oppoſite to the vnknowne angle is demaunded: or contrarily. By the two angles giuen, and a ſide oppoſite to one of them being alſo giuen: the ſide oppoſite to neither of them, or the angle oppoſite to the vnknowne ſide is required.

Theſe cannot be reſolued, but by being reduced to right angled Triangles. And they are reduced to right angled Triangles, by letting fall a perpendiculer; which perpendiculer falleth without or within the Triangle: it falleth without the Triangle if it be let fall from an acute angle: it falleth within the Triangle, if it be let fall from an obtuſe angle. Howſoeuer it falleth, it is alwayes oppoſite to the knowne angle, and is found by the third Axiom, after this manner.



1. As A D B. to A B. So is D A B. to D E.
2. As G H F. to G F. So is H G F. to H F.
3. As I M K. to I K. So is M I K. to M K.
4. As P O Q. to P O. So is O P Q. to O Q.
5. As R V S. to R S. So is V R S. to V S.
6. As W Z Y. to W Y. So is Z W Y. to Z Y.

And the perpendiculers A D, F H, K M &c. in all these obliqu-angled Triangles being found, you have two right angled Triangles, of three termes given. As for example. In the first kinde A B D and D C B. In the second E F H, and G F H, and so forwarde.

By helpe of which right angled Triangles, whatsoever is required in the obliquangled Triangles adioyning, is very easily found out, especially if every side be continued to a quadrant; after this manner.

which continuation being made,

If by A B. B C. and B D. given; I demand the first Angle.



1 As A B. to B C. So is F A. to A E. which taken from E D. there remaineth E D.

2 As H D. to D E. So is G C. to C B. whose Complement is C D. which taken from A D. there remaineth the arch A C. demanded.

But if by the same termes given, I would finde the angle A B C. I say by the second Action.

1 As D H. to H E. So is F E. to F E. which taken from E H. there remaineth F H,

2 As D H. to H E. So G C. to C B. which taken from F H. there remaineth F G. of the angle F D G demanded.

The end of the fourth Booke.

THE FIFT BOOKE OF B. P. OF THE BREIFE RVLES AND VARIETIES IN THE CALculation of Trigonometrie.

After foure former bookes I haue set downe the Rules of Trigonometrie in Triangles Rectangle: In this fifth and last booke I will shew a fountaine of breife Rules and varieties in the calculation of Trigonometrie, which although they be not of necessity, yet they are very pleasant in the vse thereof.

The breife rules in the calculation of Trigonometrie are principally, Sixe.

The first breife Rule.

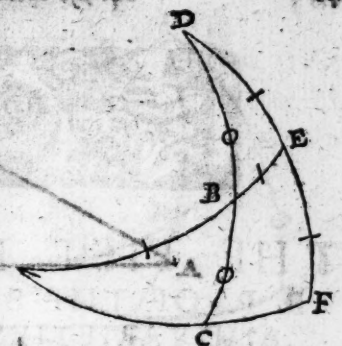
In the Rule of proportion, wherein alwaies there ought to be three tearmes giuen: If the first place be Radius and the second and third a sine, how to auoide both multiplication and diuision.

The Rule. *In stead of the two Sines giuen besides the Radius; take the Complements of the arkes, answering to those sines. And you shall haue a spherical Triangle right angled, agreeable to the fourth Axiome of spherical Triangles, and so to be resolued by* Prosthaphæricis.

As for Example. If such a proposition be giuen as Radius AE.

AE. to the ſine of EF. ſo is the ſine of AB. to the ſine of BC.

In ſtead of the giuen arches AB. and EF. in the ſecond and third place, take the Complements of them being BE. and ED; And you ſhall haue the Triangle BED, rightangled at E. by helpe whereof you ſhall finde (without any multiplication and diuiſion by the fourth Axiome) the ſide BC. demaunded, being the Complement of the ſide DB.



Then ſet the ſide AB. be 42 deg.
The ſide EF. is 48 deg. 25'.
Then the ſide BE. ſhall be 48 deg.
And the ſide DE. ſhall be 41 deg. 35'.

which thinges being thus had, I thus proceed:

DE.	41.	35'.	the ſame	41.	35'.
BE.	48.		ſ Com.	42.	
	89.	35.		83.	35. ſ ſine is 99937354
	25.				the ſine is 72721
					100100075
the ſine of the arch BC. 30 deg. 2'.					50050037

The ſecond briefe Rule.

If in the firſt place be a ſine, and the ſecond or third the Radius, to auoid diuiſion by bringing the Radius into the firſt place.

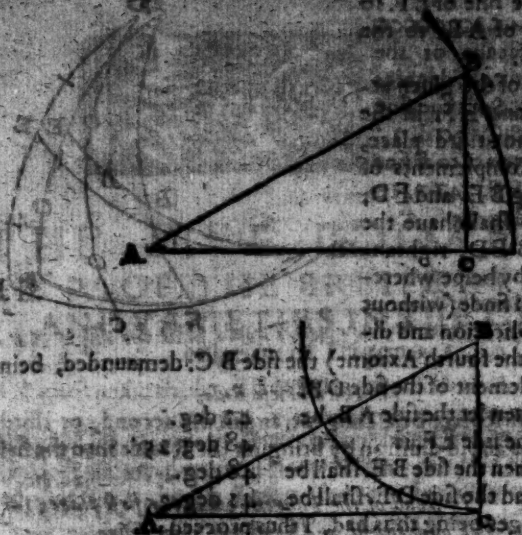
The Rule. *In ſtead of the ſine put in the firſt place, take the Secant of the Complement, ſo ſhall you haue your deſire.*

For as the ſine is to the Radius: So is the Radius to the Secant of the Complement.

As for Example.

As BC. the ſine of the angle BAC. is to AB. the Radius: So is BC. the Radius to AB. the Secant of the Complement ABC by the firſt of plaine Triangles.

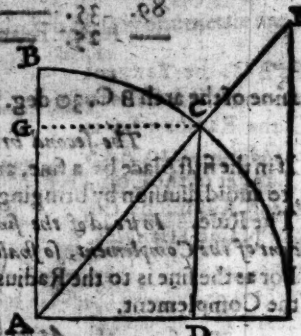
The



The same also may be thus demonstrated.

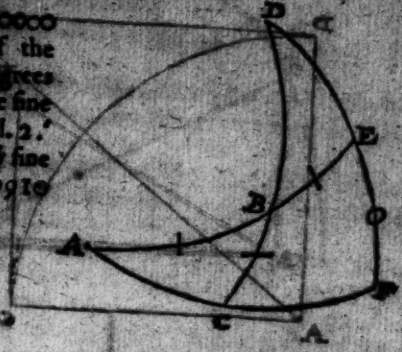
As A D. the sine of the angle A C D. to A C. the Radius: So is A F. Radius to A E. the Secant of the Complement C A D.

Example. If this proportion be given: As the sine of the arch A B. 42. deg. is to the sine of the arche B C. 30. deg. 2'. So is the sine of the arche A E. that is the Radius, to the sine of the arch E F.



In stead of the sine of the arch A B. 42 deg. take the Secant of the Complement 48 deg. and you shall have the proportion thus.

As the Radius 10000000 is to the Secant of the Complement 48. degrees 14944765. So is the sine of the arch B C. 30 d. 2.' to wit, 5005037. to the sine of the arch E F. 7479919 being 48. deg. 25'.



The third briefe Rule.

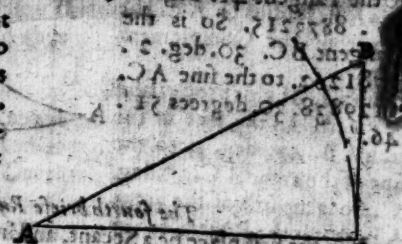
If in the first place be a Tangent, a Secant, or a Line, a Radius; to avoid division by bringing the Radius into the first place.

The Rule: *In stead of the Tangent put in the first place, take the Tangent of the Complement, and you have your desire.*

For as the Tangent to the Radius: So is the Radius to the Tangent of the Complement.

As for Example.

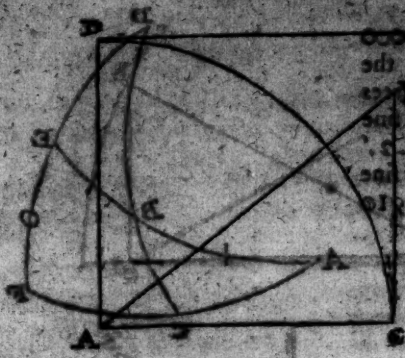
As B C. the Tangent of the angle B A C. is to A C. the Radius: So is B C. the Radius to A C. the Tangent of the Complement A B C. by the first of plaine Triangles.



The same also may be thus demonstrated.

As B C. the Tangent to A C. the Radius: So is A D. the Radius to D E. the Tangent of the Complement.

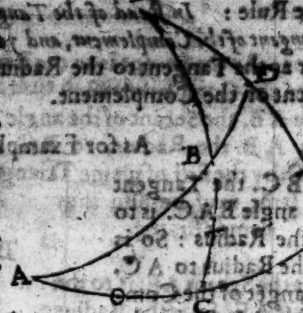
Example. If this proportion were given, as the Tangent E F. 48. deg.



48 deg. 25'. to the Radius : So is B C. the Tangent 30 deg. 2'. to the fine A C.

In ſteede of the Tangent E F. 48 deg. 25'. take the Tangent of the complemẽt 41 deg. 35'. And then your proportion will be ſuch.

As the Radius 10000000 to the Tangent 41. degrees 35'. 8873215. So is the Tangent B C. 30. deg. 2'. 5781262. to the fine A C. 5129838. 30. degrees 51'. 46".



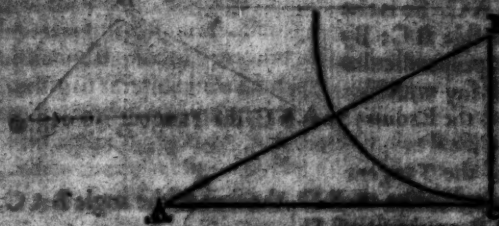
The fourth briefe Rule.

If in the firſt place be a Secant, and in the ſecond or third the Radius, to auoyde diuiſion by bringing the Radius into the firſt place.

The Rule. *In ſteed of the Secant put in the firſt place, take the fine of the Complement, and you ſhall haue a proportion wherein the Radius ſhall be in the firſt place.*

For





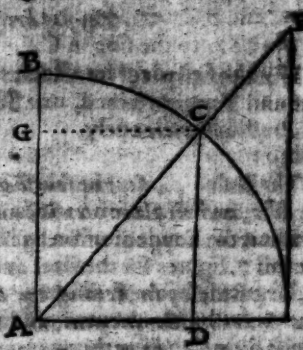
For, As the Secant is to the Radius, So is the Radius to the fine of the Complement.

As for Example:

As A B. the Secant of the angle A B C. is to B C. the Radius: So is A B. the Radius, to B C. the fine of the Complement B A C. by the firſt of plaine Triangles.

The ſame may be alſo thus demonſtrated.

As the Secant A E. is to the Radius A F. So is the Radius A C. to the fine of the Complement A D.



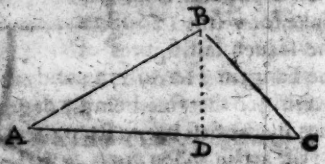
The

The fifth breife Rule.

If in the first place be a sine; and no Radius: to conuert, the diuision to be made by the first sine, into a multiplication.

The Rule. *Instead of the sine in the first place, put the Secant of the Complement, And the Problem shall be performed.*

For if such a proportion begiuen: As A B. the sine of the angle A C B. is to B C. the sine of the angle B A C. So is the side A B. to the side B C: By letting fall the perpendiculer B D. you shall say with like effect.



1 As A B. the Radius to B D. the sine of the angle B A C, So is the side A B. to the side B D.

2 As B D. the Radius to B C. the Secant of the angle D B C. being the Complement of the angle A C B. or D C B. So is the side B D. to the side B C. by the first of plaine Triangles.

The sixt breife Rule.

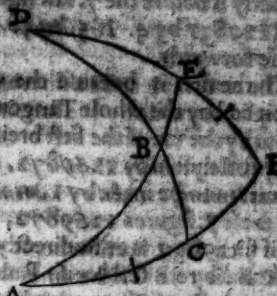
If in the first place be a Radius; and in the second and third a sine and Tangent mixed, or a Secant; to resolue the Problem be *Prosthaphæricis* onely.

The Rule. *Accompt the Tangents and Secants in the place of the sines; and the Example will be agreable to the first breife Rule.*

But if the Tangent or Secant haue more then 7. figures, take the last 7. figures for the sine, and multiply the sine of the other arche giuen, by the first figure, or figures, if their be more then one: And adde the product to the number founde by the first breife Rule, after this manner:

If such

If ſuch a proportion were giuen; As the Radius A F. to the Tangent, E F. of the angle E A F. 48 deg. 25'. which Tangent is 11269872. So is the ſine of the arche A C. (which is the arche of 30 degrees 51'. 46".) 5129837. to the Tangent of the arch B C.



Take for § ſine 1269872 the ſeauen laſt figures of the Tangent 11269872; and take out of the Table the arch anſwering to that ſine, being 7 degrees 17'. 44". Then proceed thus by the firſt breiſe Rule.

Of the arche 7 deg. 17'. 44". the Comple: is 82 deg. 42'. 16".
Of the arche 30. 51. 46. the Comple: is 59. 8. 14.

Therefore according to the fourth Axiome of ſphæricall Triangles.

The leſſer ſide is 59 d. 8'. 14", the ſame is 59 deg. 08'. 14".
The greater ſide is 82. 41. 16. § Com: is 07. 17. 44.
the ſumme is 141 d. 50'. 30". 66 d. 25'. 58". § ſine 9165916
The exceſſe 51. 50. 90. the ſine whereof is 7863064
1302852
The number found by the firſt breiſe Rule is 651426
To which adde the ſine of the other arch giuen, 5129837
The totall is the Tangent of the arch of 30 deg. 2'. — 5781263
For the arche B C. required.

Note. If the giuen Tangent were 21269872; after the order of the firſt breiſe Rule by the laſt ſeauen figures, it is 1269872: you ſhould multiply the ſine of the other arche giuen 3129837 by 2 and the product, you ſhould adde to 651426 the number found by the firſt breiſe Rule.

But if the Tangent were ſuch 31269872: after the practice of the firſt breiſe Rule by the laſt 7 figures 1269872 you ſhould multiply the ſine of th'other giuen arche 5129837 by 3.

Laſtly

Lastly if before the 7. last figures were 4. you should multiply 5129837 by 4. If 5. by 5. If 12. by 12. If 13. by 13. and so forwards.

The reason is because the whole sine 5129837 must be multiplied by the whole Tangent 1126982.

But by the vse of the first breife Rule, the sine 5129837 was onely multiplied by 1269872. Then there remained the multiplication to be made by 1. or 2. or 3. or whatsoeuer went before those 7. figures 1269872, And therefore the product by 1 and 5129837 is to be directly vnder written vnder the number found by the first breife Rule, because that found number is the product of the multiplication of the sine 5129837 and 1269872 diuided by the Radius; which product if it were not diuided by the Radius should stand thus, 6514260000000.

Then because the last multiplier 1. is in the eight place towards the left hand, therefore also the product of his multiplication 5129837 shall necessarily be so vnder written, that his last number be in the eight place, the last but one in the ninth place; and so forwards after this manner, 6514260000000,

5129837.

The seauenth breife Rule.

Whatsoeuer tearmes are giuen; to finde out the demaund by *Prosthaphæricis* onely.

The Rule. *That you may alwaies haue the Radius in the first place; Worke by the second, third, fourth, or fift breife Rule, then performe the rest by the first or sixth breife Rule.*

Of the varieties in generall of Trigonometricall calculation.

In the resolution of Triangles, especially of spheriall; one and the same demaund often times by the same thinges giuen may be found out sundry and diuers wayes: Whereof there are foure reasons, euery of which I will vnfould in seuerall Theoremes.

The first Theorem of the varietie of Trigonometricall calculation.

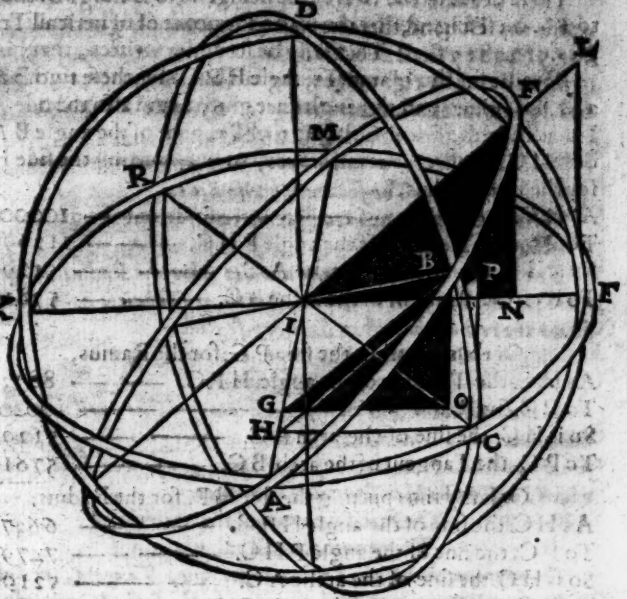
Euery proportion of the Radius to the sine, Tangent, or Secant, and contrarily may be varied 3. wayes by the first Axiome of plaine Triangles.

Therefore

Therfore in the right angled Triangle ABC. if by the arches B A C. 48. 25'. and A B. 41. deg. giuen : the arche B C. be demaunded. Because this proportion is giuen: As A E. the Radius is to E F. the sine, so is A B. the sine, to B C. the sine : By the first Axiome of sphæricall Triangles. that is,

Because in the plaine Triangle G B O, by these two giuen B A C. and A B ; all the angles, and moreouer the side G B. to wit, the sine of the arche A B. are giuen: I may finde the side B O. being the sine of the arch B C, three waies. To say either thus,

As G B. the Radius	10000000
To B O. the sine of the angle B A C: or B G O.	7479912
So is G B. the sine of the arche A B.	6691306
To B O. the sine of the arche B C.	5005038



M Or thus

Or thus.

As G B. the Secant of the angle B G O. —— 15066852
To B O. the Tangent of the same angle, —— 11269872
So is G B. the sine of the arche A B. —— 6691306
To B O. the sine of the arche B C. —— 5005038

Or lastly thus.

As G B. the Secant of the angle G B O. —— 13369141
To B O. the Radius, —— 10000000
So is G B. the sine of the arche A B. —— 6691306
To B O. the sine of the arche B C. —— 5005038

So in the same spharicall Triangle A B C. If by B A C. 48 deg. 25'. and A C. 30 deg. 51'. 40". giuen: the arche B C. be demaunded. Because the proportion is giuen thus

As *A F.* the Radius to *E F.* the Tangent, so is the *A C.* the sine to *B C.* the Tangent: by the second Axiome of spharicall Triangles. That is,

Because in the plaine Triangle H P C. by these two B A C. and A C. giuen; all the angles are giuen; and also the side A C: I may finde the side P C. being the Tangent of the angle B A C. or P H C. by three seuerall wayes, to wit, putting the side H C. for the Radius thus.

As H C. the Radius —— 10000000
To P C. the Tangent of the angle P H C. —— 11269072
So is H C. the sine of the arche A C. —— 5129838
To P C. the Tangent of the arche B C. —— 5781262

Or thus, putting the side P C. for the Radius,

As H C. the Tangent of the angle H P C. —— 8873215
To P C. the Radius —— 10000000
So is H C. the sine of the arch A C. —— 5129838
To P C. the Tangent of the arch B C. —— 5781262

Or lastly thus putting the side H P. for the Radius.

As H C. the sine of the angle H P C. —— 6637087
To P C. the sine of the angle P H C. —— 7479912
So is H C. the sine of the arche A C. —— 5219838
To P C. the Tangent of the arche B C. —— 5781262

The

The second Theorem of the variety of Trigonometricall calculation.

In the rule of Proportion, wherein there are alwaies foure rearmes; three giuen, the fourth demaunded: It is all one whether of the two middle rearmes I shall put in the second or third place.

For it is all one whether I shall say,

As 2. to 4. so 5. to 10. Or,

As 2. to 5. so 4. to 10.

From hence euery Example of the first Theorem may be varied three wayes.

The first Example of the first Theorem was thus,

As the Radius,	10000000
To the sine of the angle B A C.	7479912
So is the sine of the arche A B.	6697306
To the sine of the arche B C.	5005038

In stead thereof I may now say, vsing againe the variety of the first Theorem.

As the Radius,	10000000
To the sine of the arche A B.	6691306
So is the sine of the angle B A C.	7479912
To the sine of the arche B C.	5005038
Or as the Secant of the arche A B.	13456327
To the Tangent of the same arche,	9004040
So is the sine of the angle B A C.	7479912
To the sine of the arche B C.	5005038
Or lastly, As the Secant of ½ Comple: of ½ arch A B.	14944765
To the Radius,	10000000
So is the sine of the angle B A C.	7479912
To the sine of the arche B C.	5005038

The second Example of the first Theorem was thus:

As the Radius,	10000000
To the Tangent of the angle B A C.	11269872
So is the sine of the arche A C.	5129838
To the Tangent of the arche B C,	5781262

In stead thereof I will now say: vsing the variety also of the first Theorem.

As the Radius,	10000000
To the sine of the arche A C.	5129838

So is the Tangent of the angle B A C. —— —— 11269872
To the Tangent of the arche B C. —— —— 5781262

Or, As the Secant of the arche *A C.* —— —— 11649603
To the Tangent of the same arche —— —— 5976055
So is the Tangent of the angle *B A C.* —— —— 11269872
To the Tangent of the arche *B C.* —— —— 5781262

Or lastly, As the Secant of the Comp: of ½ arch A C. 19493797
To the Radius, —— —— —— —— —— 10000000
So is the Tangent of the angle B A C. —— —— 11269838
To the Tangent of the arche B C. —— —— 5781262

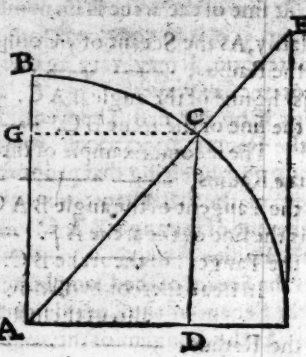
The third Theorem of the variety of Trigonometricall calculation.

The sines of the Arches and the Secants of the Complements are reciprocally proportionall.

That is, As the sine of the greater arche is to the sine of the lesser arch: So is the Secant of the Complement of the lesser arch to the Secant of the Complement of the greater arche.

And in like manner, As the sine of the lesser arche is to the sine of the greater arche: So is the Secant of the Complement of the greater arche to the Secant of the Complement of the lesser arche.

The reason of this reciprocall proportion, is *Because the Radius is a meane proportionall betweene the sine of any arch and the Secant of the Complement of that arche.* That is, As the sine is to the Radius. So is the Radius to the Secant of the Complement. As for Example,



As *A D.* the sine of the arche *B C.* is to the Radius *A C.* So is the Radius *A E.* to *A E.* the Secant of the Complement *C F,* by the fourth of the sixth of *Euclide*; Or by the 46. of the first hereof.

Therefore whatsoeuer sine is multiplyed by the

Secant

Secant of the Complement, it maketh the ſquare of the Radius.

And thereupon, the plaine figures made of the ſines of the arches, and the Secants of the Complements, are all equall to one another: viz. they are equall to one, and the ſame ſquare of the Radius.

But equall plaine figures haue their ſides reciprocally proportionall by the 42. of the firſt. Therefore as the ſine of a greater arch is to the ſine of any leſſer arch: So is the Secant of the Complement of the leſſer arch to the Secant of the Complement of the greater arch.

All this is very eaſie to be diſcerned in ſmall numbers: For let the two ſines be 4. and 2. the Radius 10.

Firſt, it is manifeſt that the Secants of their Complement are 25. & 50. For,

As 4. to 10. ſo is 10. to 25. and
As 2. to 10. ſo is 10. to 50.

Then it is manifeſt, that the Secant of the Complement 2. is to the Secant of the Complement 4. As 4. to 2. For the Secant the Complement of 2. is 50. And the Secant of the Complement of 4. is 25. Then as 4. is to 2. So is 50. to 25.

In greater numbers is the ſame reaſon. For let the two ſines giuen, be 6691306. and 5005038. and let the Secants of the Complements be demaunded after this manner.

As 6691306. to 10000000. ſo is 10000000. to 14944765. &
As 5005038. to 10000000. ſo is 10000000. to 19979868.

It is manifeſt after theſe numbers found, that

As 6691306. is to 5005038. ſo is 19799868. to 14944765.

Hereupon, I may vary againe the firſt example of the firſt Theorem, ſixe wayes.

For if by the firſt Theorem, I ſhall take this proportion: As the Radius 10000000. to the ſine of the angle B A C. 7479912. inuerting that proportion, I may ſay, vſing alſo the variety of the firſt Theorem: either,

As the ſine of the angle B A C. ——— ——— 7479912.
To the Radius ——— ——— ——— ——— 10000000
So is the Secant of the Complement of the arch A B. 14944765
To the Secant of the Complement of the arch B C. 19979868
Or. As the Tangent of the angle B A C. ——— 11269872

To the Secant of the same angle ——— 15056852
So is the Secant of the Complement of the arch AB. 14944765
To the Secant of the Complement of the arch BC. 19979868
Or lastly, as the Radius ——— 10000000
To the Secant of the Complement of the angle BAC. 13369141
So is the Secant of the Complement of the arch AB. 14944765
To the Secant of the Complement of the arch BC. 19979868

But if by the Second Theorem, I shall take this proportion.

As the Radius 10000000, to the sine of the arch AB. 6691306

inuerting that proportion, I may say by this third Theorem, vsing the variety of the first Theorem in like manner; either,

As the sine of the arch AB. ——— 6691306
To the Radius ——— 10000000
So is the Secant of the Comple. of the angle BAC. 13369141
To the Secant of the Complement of the arch BC. 19979868

Or, As the Tangent of the arch AB. ——— 9004040
To the Secant of the same arch ——— 13486377
So is the Secant of the Comple: of the angle BAC. 13369141
To the Secant of the Complement of the arch BC. 19979868
Or, Lastly, as the Radius ——— 10000000
To the Secant of the Complement ——— AB. 14944765
So is the Secant of the Comple: of the angle BAC. 13369141
To the Secant of the Complement of the arch BC. 19979868

So by the same BAC. and AB. giuen, I shall finde the arch BC. 12. wayes thrice by the first Theorem, and againe thrice by the second, and lastly sixe times by the third Theorem.

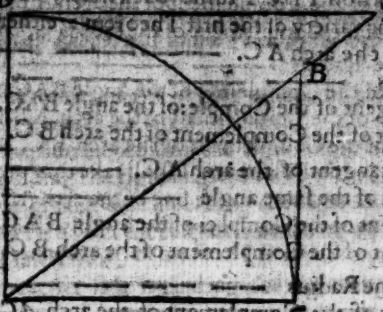
The fourth Theorem of the variety of Trigonometricall calculation.

The Tangents of arches, and the Tangents of the Complements are reciprocally proportionall.

That is, as the Tangent of the greater arch is to the Tangent of the lesser arch: So is the Tangent of the Complement of the lesser arch; to the Tangent of the Complement of the greater arch. And contrarily,

The reason of this reciprocall proportion is, the same that is

in



in the Secants; that is because the Radius is a meane proportion betwixt the Tangent of an arche, and the Tangent of the Complement. For,

As E D. to A D. So is A C to B C. by the 4. of the first of *Euclide*, or by the 46 of the first hereof.

As for example: When the proportion is,

As 11269872. the Tangent of 48 deg. 25'. to the Radius 10000000, So is the Radius 10000000. to 8873215. the Tangent of the Complement. And,

As 5781262. the Tangent of 30 deg. 2'. to the Radius 10000000. so is the Radius 10000000. to 17297260. the Tangent of the Complement. It shall be also,

As 11269872. to 5781262. so is 17297260. to 8873215

Or Contrariwise,

As 5781262. to 11269872. so is 8873215. to 17297260.

Hereupon, if by 11269872. the Tangent given, the Tangent 5781262. be demanded, leaving those Tangents, I may suppose 8873215. the Tangent of the Complement to be given, and the Tangent of the other Complement 17297260. to be demanded. In taking of which supposition, I inuert the proportion of the second example of the second Theorem; which was thus: As the Radius 10000000. to the sine of the arch A C. so is the Tangent of the angle B A C. to the Tangent of the arch B C.

This proportion I say I turne backward, and say, vsing therewithall the variety of the first Theorem: either,

As the sine of the arch A C. ——— 5129838
To the Radius ——— 10000000
So is the Tangent of the Comple: of the angle B A C. 8873215
To the tangent of the Complement of the arch B C. 17297260

Or, As the tangent of the arch A C. ——— 5976055
To the Secant of the same angle ——— 11649603
So is the tangent of the Comple: of the angle B A C. 8873215
To the tangent of the Complement of the arch B C. 17297260

Or lastly, As the Radius ——— 10000000
To the Secant of the Complement of the arch A C. 19493797
So is the tangent of the Comple: of the angle B A C. 8873215
To the tangent of the Complement of the arch B C. 17297260

And so by the same B A C. and A C. giuen, I shall finde the arch B C. nine wayes, thrice by the first Theorem; thrice by the second, and againe thrice by the fourth Theorem.

Touching the varietie of Trigonometricall calculation in perusing: concerning the three former Axiomes of plaine Triangles.

The three former Axiomes of plaine Triangles, may happily be more rightly drawne into one, and may thus be propounded.

The sides are directly proportionall, to the subtenses of the opposite angles.

That is, as the greatest side is in proportion to the least side: So is the subtense of the greatest angle in proportion to the subtense of the least angle. And so of the rest.

The reason is, because a circle may be circumscribed to euery plaine Triangle: which if it be done, then the sides themselues of the plaine Triangle, are the subtenses of the angles opposite thereunto.

A generall Conselusie.

Therefore the subtenses being giuen, of whatsoeuer two angles, with a side opposite to one of the angles giuen: the side also opposite to the other of the giuen angles is giuen: And contrarily,

The

The two sides whatsoever being given, with the subtense of any angle opposite to one of those sides given: the subtense also of the angle opposite to the other side, is also given: and by the subtense thereof the angle it selfe.

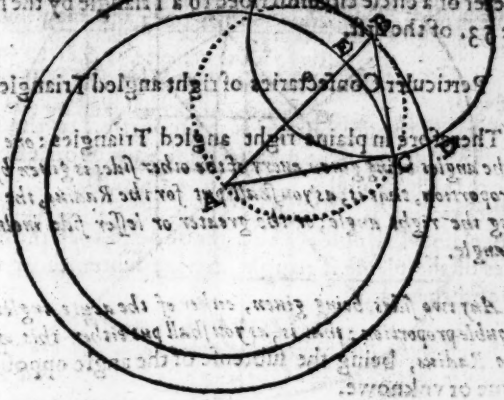
And the subtenses of the angles giuen, in plaine Triangles, are giuen three wayes, to wit, either thus,

1. That the side subtending the right angle be Radius, and the sides including the right angle, sines. Or thus,

2. That the greater side including the right angle be Radius, and the other two sides the tangent and secant of the lesser acute angle. Or lastly thus,

3. That the lesser side including the right angle be Radius. And the other two sides, the tangent and secant of the greater acute angle.

As in the plaine right angled Triangle *ABC*, wherein the sides *AB*, *BC*, and *AC*, are the subtenses of the angles opposite vnto them, in respect of the pricked circle *BC*: if you put the side *AB*, for the Radius; the sides *BC*, and *AC*, shall be the sines of the angles *BAC*, and *ABC*, in respect of the Circle *BD*.



If you put the side *A C.* for the Radius, the side *B C.* shall be the Tangent of the angle *B A C.* and the side *B A.* shall be the Secant of the same angle in respect of the circle *B C.*

If you put the side *B C.* for the Radius, the side *A C.* shall be the Tangent of the angle *A B C.* and the side *A B.* shall be the Secant of the same angle, in respect of the circle *C D.*

But in plaine obliquangled Triangles, the angles being giuen, the subtenses are giuen by one way, to wit by the first one: For the Tangents and Secants in plaine obliquangled Triangles are of no vse by their definitions.

But the sines are of vse in all, because they are the halfe of the subtenses of the double circle, which subtenses of euery plaine Triangle may be made the sides, by the demonstration aforegoing.

But in a plaine right angled Triangle the subtense of euery angle cannot be giuen: For euery side of a plaine right angled Triangle may be put for the Radius, that is, for the Diameter of the circle circumscribed to the Triangle, and so may be accompted for the subtense of the angle opposite, not yet knowne.

But in a plaine obliquangled Triangle, the subtense of an annot giuen, can by no meanes bee giuen: Because no side of a plaine obliquangled Triangle can be put for the Radius; and that because no side of a plain obliquangled Triangle can be the Diameter of a circle circumscribed to a Triangle by the first *Con:* of the 53. of the first.

Perticuler Confectaries of right angled Triangles.

1 Therefore in plaine right angled Triangles: *one side, besides the angles being giuen, euery of the other sides is giuen by a three-fold proportion, that is, as you shall put for the Radius, the side subtending the right angle; or the greater or lesser side including the right angle.*

2 *Any two sides being giuen, either of the acute angles is giuen by a double proportion: that is, as you shall put either this or that side for the Radius,* being the subtense of the angle opposite, either knowne or vnknowe.

Perticuler

Perticuler Conſectaries of obliquangled Triangles.

1. In plaine obliquangled Triangles, *one ſide being giuen beſides the angles, many of the other ſides may be giuen, but by only two proportions.*

2. Any two ſides being giuen, not ſimply, *but only two ſides being giuen with an angle oppoſite to one of them, the angle oppoſite to the other of them is giuen.*

This abſtract in my opinion was more methodicall, but that reaſon, which I haue laid downe in my third booke for the vnderſtanding of young learners, was more fit, in the opinion of thoſe my Schollers, who had ſome intereſt in me.

Againe: Of the varieties of trigonometricall calculation in perticuler.

About the fourth Axiome of Sphericall Triangles.

Regiomontanus, & *Finckius*, and *Landſbergius* following him, doe thus propound the fourth Axiome of ſphæricall Triangles.

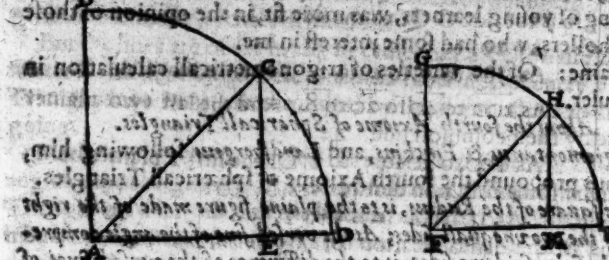
The ſquare of the Radius, is to the plaine figure made of the right ſines of the two vnequall ſides; As the verſed ſine of the angle comprehended of the ſaid two ſides, is to the difference of the verſed ſines of the third ſide, and the difference of the other two ſides.



Their

Their demonstration is briefely thus.

1 As *G H* the Radius to *D E* the right sine of the greater side *B C* or *B D*, So is *G O* the versed sine of the angle *A B C* in the diameter of a great Circle, to *D C* the same versed sine in the diameter of the circle parallell: Because in vnequall Circles: *As the Radius of one Circle to the Radius of another Circle: So is the sines aswell right as versed of the one Circle, to sines of like arches, aswell right as versed of the other Circle.*



As for example.

In the vnequall Circles *B C D* and *G H I*: If *C D* and *H I* be like arches. Then.

As the Radius *A C* to the Radius *I H*: So is the right sine *C E* to the right sine *H K*; and so is the right sine *A E* to the right sine *F K*: And lastly so is the versed sine *E D* to the versed sine *K I* by the fourth of the sixt, and by the first of the fift, & the eleauenth of the seauenth booke of *Euclide*, and by the Schemes adioyning.

2 As *G H* the Radius to *G O* the right sine of the lesser side *A B* or *G N*: So is *D C* to *D L* by the fourth of the sixt booke of *Euclide*: which *D L* being added to *A F* the versed sine of the difference of the sides *A B* and *B C* or *B D*; to wit, of the arch *A D*; maketh *A M* the versed sine of the third side *A C* or *A K*. Moreouer, because

As *G H* to *D E* so is *G Q* to *D C*. And

As *G H* to *G O* so is *D C* to *D L*. It is also by the multiplication of proportions.

As

As the plaine figure G H G H. to the plaine D E G O. So is the plaine G Q D C. to the plaine D C D L. And the last two plaines G Q D C. and D C D L. being diuided by their common side D C.

As the plaine G H G H. to the plaine D E G O. So is the side G Q. to the side D L.

Or the first two plaines also be deuided by some common diuisor, to wit, the Radius.

As G H the Radius to the plaine D E G O. diuided by the Radius: so is the side G Q. to the side D L.

For if: As 10. to 8. so 5. to 4. And,

As 10. to 5. so 4. to 2. Then it shall be,

As 100 to 40 so 20 to 8. And the last two plaines diuided by their common side 4:

As 100 to 40 so 5 to 2. Or the first 2 plaines being diuided by some common diuisor: viz. by 10:

As 10 to 4 so is 5 to 2.

This is the demonstration of *Regiemontanus Finckius* and *Landsbergius* altogether certaine and infallible. Which no man can well vnderstand that is not a sufficient Geometrician.

An Example, repeated out of the third booke of my Examples.

B C. 59 deg. 58'.	the right sine is	8657344
A B. 45. 58.	the right sine is	7189355
Difference 14.		10000000
76.		9702957
The versed sine of the difference		00297043
A B C. 28 deg. 14'.		10000000
61. 46.		8810284
The versed sine of the angle		1189716
The plaine made of ỹ right sines A B & B C.		6224071 9373120
The same plaine deuided by the Radius, is		6224072

Truely agreing with the halfe of the right line found of me, by *Prosthaphæricis*.

The proportion.

As the Radius	10000000	D E.
To ỹ plaine of ỹ right sines diuided by ỹ Radius	6224072	D T.
So is the versed sine of the angle A B C.	1189716	D C.
To the difference	740488	D L.

which

Which if you add to the verſed ſine of the diffe-} 297043 A F.
rence of the ſide, ——— }

Maketh the verſed ſine of the third ſide ——— 1097531 A M
Which taking from the Radius ——— ——— 10000000 A H.
Leaueth the ſine of the Complet of ſ third ſide 8962469 M H

To which ſine the arch *K N.* 63 degrees 40'. 8". anſwereth the Complement whereof is *A K* or *A C* 26 deg. 20'. 52". the arche demaunded.

Iuſtus Bergius *in the working of the fourth Axiome, neuer vſeth the verſed ſines, but alwaies the right ſines.*

And firſt, The angle at B. being a right angle, hee ſeeketh out what ſhould be the ſine of the Complement of the third ſide: then if the angle at B, be acute, he findeth the difference of that ſine from the ſine of the oblique angle, that is, the right line C O. or E O. or L T. by ſuch proportion.

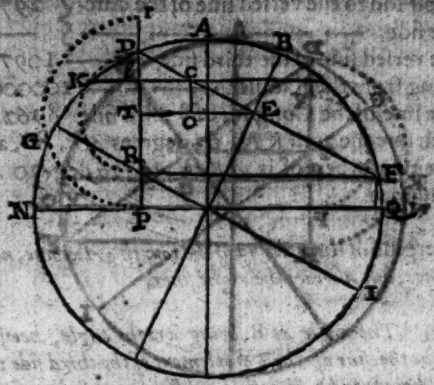
As D E. the Radius to D T. the halfe of the right line, So is C E the ſine of the Complement of the angle A B C. to C O.

If the angle at B be obtuſe; by ſuch proportion: As D E. the Radius to D T. the halfe of the right line, ſo is E C. the ſine of the exceſſe of the angle A B C. to E O.

Which proportion notwithſtanding; he reſolueth the ſame without any multiplication and diuiſion, by the help of my firſt briefe Rule.

Moreouer, the angle A B C. being acute, he alwayes addeth L T the number found to T P. to make L P. the ſine of the Complement of the third ſide. But the angle A B C being obtuſe, either he ſubſtracteth the found number T L, from T P. that there may remaine L P, the ſine of the Complement of the third ſide; or els he ſubſtracteth T P. from the found number T L. that the remainer may be L P. the ſine of the exceſſe of the third ſide: all which the three ſchemes following do teach, in euery of which, I haue ſet downe an Example after Birgius *his way.*

The



The first example, which was the second of the second kinde in the fourth booke.

The lesser side G N. or Q I. 26 deg. 20′. } Adde and
The Com: of ỹ greater G D. } 30. 02. } Substract
Or F I. }

The summe is D N. 56. deg. 22. the sine 8325991. D P.
The difference F Q. 3. 42. the sine 645323. P R.

The summe is 8971314. r P.
The $\frac{1}{2}$ is the first found number 4485657. T P.
645323. D r.

22 deg. 35′. I $\frac{1}{2}$″. is the arch of 3840334. D T. the $\frac{1}{2}$ of the right line

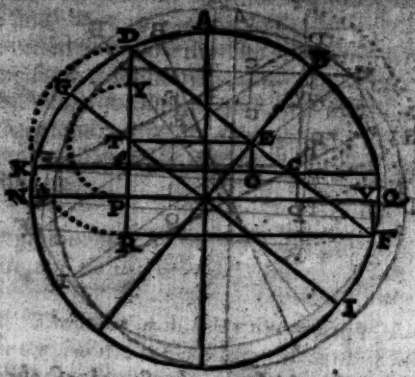
50. 04. 0. put for A B C. as it were the lesser side.

72. 39. I. $\frac{1}{2}$ the sum thereof, the sine is 9545031
27. 28. 58. $\frac{1}{2}$ their difference, the sine is, 4614841

Their difference is, 4930190
the second number found is the $\frac{1}{2}$ thereof 2465095. CO. or L T
the first number found, was —— —— 4485657. T P.

the totall is, 6950752. LP. the sine of the Complement of the third side.

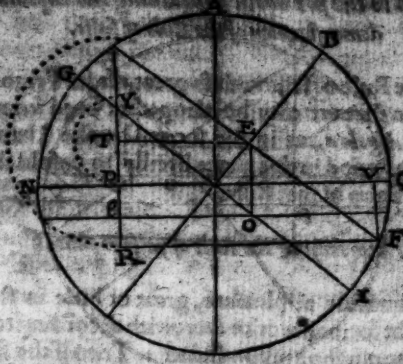
The



The second Example, which was the third of the third kinde in the fourth booke.

The lesser side G N. or 21. ——— ——— 45 d. 58′.
The Comple: of the greater side G D. or F I. 30. 02.
The summe is D N. 76. —
Summe is, D N. — 76 d. 19′. the sine 9702957. D P.
the differ: F 2. is 15. 56. the sine 2745187. P R. or D Y
the difference is 6957770. P Y.
the ½ is, —— 3478885. I P. & 1 foūd n°
2745187. P R. or D Y.
38 d. 29′. 31″. 6224072. D T. ½ of the
right line. ——— ———
The lesser side —— 51. 30. 29
The excesse A B C. 22. 35. 00.
The summe is 74. 05. 29. the sine 9617001
The differeuce 28. 55. 29. the sine 4836600
the difference 4780401
the ½ is —— 2390200. E O. or
L T. the 2. found N°. ——— 3478885. I P. the 1.
found N°. the difference is 1088685. L P. being
the sine of the Complement of the third side.

The



The third Example which was the fourth of the third kinde in the fourth booke,

The leſſer ſide G N. or *Q I.* 45 d. 58'.
The Comp: of the greater ſide G D. or F I. } 30. 02.
The ſumme is D N. 76. — the ſine 9702957. D P.
The difference *Q F.* 15. 56. the ſine 2745187. P R. or D Y.
the difference 6957770. Y P.
the ½ is — 3478885. T P. & 1. ſoud N°
2745187. P R. to D Y.
38 d. 29'. 31". 6224072. T R. the ½ of the right line.
ABC. as the leſſer ſide 10. — —
The ſumme 48. 29. 31. & ſine 7488925
The differēce 28. 29. 31. the ſine 4770352
The ſumme 12258977
The ½ is the ſecond found N°. 6129488. E O. or T L.
the firſt found N°. 3478885. *T P.*
The difference 2650603. *P L.* the ſine of exceſſe of the third ſide.

This is Bergius *his way: to ſpeake of other waies it is not worth the labour.*

An addition to the fift booke containing the explaining and demonſtration of the Rule of falſe.

Becauſe the Rule of falſe hath ſo great vſe in Trigonometria, as a Scholler in that art may be altogether freed from the intricate Rules of *Algebra*, as in the ſecond booke I haue ſhewed; I haue thought good in this place breifly to vnfold the precept and demonſtration of that Rule.

The precept of the Rule of falſe is thus:

Take any number at pleaſure, great or little in ſtead of the number ſought for; and worke therewith according to the order or nature of the queſtion propounded: Then if the facit or anſwere be juſt as it ought to be, you haue your deſire: But if otherwiſe, Note the difference or error by + or — And by an other poſition either greater or leſſer, then the firſt, repeat the former worke, and likewiſe note the error by the ſaid more or leſſer, After multiply alternatly by a croſſe, the firſt poſition by the ſecond error, and alſo the ſecond poſition by the firſt error. Then if the Errors haue like ſines, ſubſtract the leſſer product from the greater, and likewiſe the two errors the one from the other: But if the two errors haue vnlike ſines, adde the two products together and alſo the two errors.

And laſtly diuide the totall or the remainder of the two products by the totall or remainder of the two errors, the quotient is the true number ſought for.

Therefore in this Rule there are three caſes.

1 The firſt where both the errors are +.
2 The ſecond where both the errors are —.
3 The third where the one is + and the other leſſe.

Example.

What number is that to which if I adde $\frac{1}{3}$ thereof, and from the totall ſubſtract $\frac{1}{4}$ of the whole the remainder is 100.

The

The true number sought is 90 as appeareth by the worke.

90	the number supposed.
30	the ⅓ thereof.
120	the summe added,
020	the ⅙ substracted from thence,
100	Remaineth.

But imagine I know not the number sought for, And for the first case. I will first suppose that number to be 144: And after I will suppose it, to be 108. As appeareth by the worke following:

1 *Position* 144	2 *Position* — 108
⅓ 48	⅓ 36
192	144
⅙ 32	⅙ 24
160 — *The facits* —	120
100 — *The true facit* —	100
1 *Error* + 60	*The* 2. *Error* + — 020
2 *Position* — 108	*The* 1. *Position* — 144
6480	*The products* — 2880
2880	
Resteth — 3600	*the dividend. The* 1 *Error* + 60
40	*the divisor. The* 2 *Error* + 20
90	the quotient *Resteth* — 40 *The divisor*

or number sought for.

For the second case I will suppose the number sought for; First 54. and after 72: As by the worke Following is manifest.

1 *Position* 54	2 *Position* 72
⅓ 18	⅓ 24
72	96
⅙ 12	⅙ 16
60	80
100 The true facit —	100

1 *Error* — 40 2 *Error* — 20

2 *Position* — 72 1 *Position* — 54

2880 1080

1080

1800 the dividend. 1 *Error* — 40

20 the divisor. 2 *Error* — 20

(90 the quotient *the divisor* 20

or number sought for.

3 For the third case. I will first suppose the number required to be 54 : And after 144.

The Worke.

1 *Position* 54 2 *Position* — 144

$\frac{1}{3}$ 18 $\frac{1}{3}$ 48

72 192

$\frac{1}{5}$ 12 $\frac{1}{5}$ 32

60 160

100 100

1 *Error* — 40 2 *Error* + — 60

2 *Position* 144 1 *Position* — 54

5760 3240

3240

9000 *the dividend.* 1 *Error* — 40

100 *the divisor.* 2 *Error* + 60

100 *divisor.*

(90 the quotient

or number sought for.

The ground of which Rules is thus : That the *Errors* or falsities of the Positions : And of the Numbers found are proportionall one to another : That is, As the error of the first position is to the error of the second position ; So is the error of the first found number, to the error of the second found number : I call the error of Positions the excesse or defect of the Numbers supposed

poſed, aboue or vnder the true number ſought for. As in the firſt caſe.

1 *Poſition*	144	2 *Poſition*	108
The true number	090	*The true number*	90
The difference	054	*The difference*	18

Therefore the error of the firſt poſition was 54
And of the ſecond 18

The errors of the Numbers found I call the exceſſe or want of the numbers by the worke produced, either more or leſſe then the number to be produced: As in the firſt caſe.

1 Found number	160.	2 Found number	120
	100	The number to be produced	100
	60	The differences	20

Therefore the error of the firſt found number was 60, And of the ſecond 20. Therefore As 54 to 18: So is 60 to 20. The reaſon, Becauſe the worke in both poſitions was after the ſame manner: that is, by adding to the firſt ſuppoſed number $\frac{1}{3}$ thereof, And (from that totall) by taking away $\frac{1}{3}$ thereof.

The effect is anſwerable to the reaſon as by the worke following.

As 54 to 18. So 60 to 20.
Multiply 18.
By 60.

The product is 1080, which diuide by 54, the quotient is 20.

Becauſe therefore: As the error of the firſt poſition to the error of the ſecond poſition: So is the error of the firſt found number, to the error of the ſecond found number.

Therefore if thoſe errors be multiplied alternatly or by the croſſe, that is the error of the firſt poſition, by the error of the ſecond found number; And the error of the ſecond poſition, by the error of the firſt found number; The product of thoſe two multiplications ſhall be equall. For if there be foure numbers proportionall, the product of the two meanes ſhall be equall to the product of the two extreames.

Seeing therefore the proportion is

As 54 to 18. So is 60 to 20.

The product of the number 18 by 60 shall be the same with the product of 48 by 20 which the worke following shall make manifest.

The error of the first position	54.	
The error of the second position	18	
The error of the second number found	20	
The error of the first found	60	
	1080.	1080

Moreouer in multiplication it is all one, whether I multiply the whole number by the whole, or one whole number by the parts of an other: As for Example It is all one whether I multiply 7. by 7; or 7. by 4. and 3. For by both multiplications I shall finde 49. as reason teacheth, and the worke following sheweth,

7	7	7
7	4	3
49	28	21

Add 21

Fa: — 49

Therefore the Example propounded in the first case: If I multiplye the first position 144. by the Error of the second number found viz. by 20. It is all one as if I should haue multiplied 90. and 54. by 20. And consequently the product of the multiplication of the numbers 144. by 20. containeth the true number 90. 20 times; and the error 54. likewise 20 times.

1. In like manner, If I multiplie the second position 108. by the error of the first found number, that is, 60. It is as much as if I should multiplie 90. and 18. by 60. And consequently the product of the multiplication of the number 108. by 60. conteyneth the true number 90. 60 times; and also the error 18. 60 times.

But 18 taken 60 times & 54. 20. times are equall. Therfore, if from the product of 108. multiplied by 60. I substract the product of 144. multiplied by 20. I then shall substract the error altogether

altogether, that came out with the firſt product: And alſo I ſhall ſubſtract the true number 20 times; And the Remainder ſhall conteyne the true number 40 times, that is, as many times as the Remainder ſhall be after the ſubducting of the error 20. from the error 60.

Therefore if I diuide that which remaineth after the ſubſtraction of the one product from the other, by the Remainer of the errors of the 2. numbers found, one error being ſubſtracted from an other; The Quotient of neceſſitie muſt be the true number.

2 Againe, after the ſame manner in the ſecond caſe: If I multiplie the firſt poſition 54. by the ſecond error 20. it is as much as if I ſhould multiply 20. by 90. leſſe 36. Then the product of 54. by 20. containeth 90. the true number 20. times, leſſe by the Error 36. 20. times.

And in like manner, if I multiply the ſecond poſition 72. by the firſt error of the number found viz. 40. It is as much as if I ſhould multiply 90. leſſe 18. by 40. Then the product of 72. by 40. containeth the true number 90. 40. times: leſſe alſo by 40. times 18. the error or difference from the true number 90. Againe, 40. times 18. and 20 times 36. are equiualent as aforeſaid; Therefore if I ſubſtract the product of 54. by 20 from the product of 72. by 40. I wholy ſubſtract the error produced in the firſt product; and alſo the true number 20. times Then the remainder ſhall be the true number 20. times. As the Error 20. ſubſtracted from 40. there remaineth 20.

Therefore if I deuide the remainder of the 2. products by the remainder of the 2 errors of the numbers found: The quotient ſhall be the true number demanded.

3 In the third caſe, if I multiply the firſt poſition 54. by the error of the ſecond found number viz, 60. It is the ſame, as if I ſhould multiply 90. leſſe 36. by 60, then the product ſhall containe 90. the true number 60. times, leſſe by 60. times the error 36.

If I multiplie the ſecond poſition 144. by 40, it is all one, as if I ſhould multiply 90. and 54. by 40. therefore the product ſhall containe the true number 90. 40. times, and alſo the error 54. 40. times.

But

But 60. times 36. and 40. times 54. are equall in power one to another as afore: And therefore, what is wanting in the one place, is ouer in the other: And consequently, if I adde the product of 54. by 60. to the product of 144. by 40. the totall shall be no more a false number, but shall containe 90. the true number, 60. times, and 40. times; that is 100. times.

Therefore, if I deuide the totall of the products, by the totall of the errors of the found numbers, I shall haue the true number required.

The end of the fift booke.

FINIS.

QVESTIONS OF NAVIGATION, PERFORMED ARITHMETICALLY BY THE DOCTRINE of Triangles, without Globe, Sphære, or Mappe.

Written by RAPHE HANDSON.

Wherein is manifested, *The disagreement betwixt the ordinarie Sea-Chart, and the Globe, And the agreement betwixt the Globe, and a true Sea-Chart made after Mercators way, or Mr.* Edw: Wrights *proiection: Whereby the excellency of the Art of Triangles will be the more perspicuous.*

THe meridians in the ordinary Sea-Chart are right lines, all parallel one to another, and consequently doe neuer meet: Yet they cut the Equinoctiall and all circles of latitude or parallels thereunto, at right angles, as in the terrestriall Globe: but herein it differeth from the Globe; for that here, all the parallels to the Equinoctiall being lesser Circles, are made equall to the Equinoctiall it selfe, being a great Circle, and consequently the degrees of those parallels or lesser circles, are equall to the degrees of the Equinoctiall, or any other greater circle, which is meerely falſe, and contrary to the nature of the Globe, as shall be hereafter more plainly demonstrated.

The Meridians in the terrestriall Globe, doe all meet in the

poles of the world, cutting the Equinoctiall, and conſequently all circles of latitude or parallels to the Equinoctiall at right ſphæricall angles; So that all ſuch parallels, doe grow leſſer towards either pole, decreaſing from the Equinoctiall line. As for example: 360. deg. or the whole circle of the parallel of 60 deg. is but 180 deg. of the Equinoctiall, and ſo of the reſt; whereas in the ordinary Chart, that parallel and all other are made equall one to an other, and to the Equinoctiall Circle, as before ſaid.

The Meridians in a Mappe of Mr. *Wrightes* proiection, are right lines all parallel one to another, and croſſe the Equinoctiall, and all circles of latitude at right angles, as in the ordinarie Chart: but here though the Circles of latitude are all equall to the Equinoctiall, and one to another, both wholy and in their parts or degrees; yet they keepe the ſame proportion one to another, and to the Meridian it ſelfe, by reaſon of the inlarging thereof, as the ſame parallels in the Globe doe: wherein it differeth from the ordinary Chart. For that there the degrees of the Meridian, and the degrees of all circles of latitude are equall: And here, though the degrees of all circles of latitude are equall, yet are the degrees of the Meridian vnequall, being inlarged from the Equinoctiall towards either pole to retaine the ſame proportion as they doe in the Globe it ſelfe: For as two degrees of the parallel of 60. is but one degree of the Equinoctiall or of any great circle vpon the Globe: So here, two degrees of the Equinoctiall, or of any circle of latitude, is but equall to one degree of the Meridian betwixt the parallels of 59 and 60, and ſo forth of the reſt.

Alſo their agreement may be thus farther manifeſted.

Such proportion as one Circle hath to another; ſuch proportion have their degrees, Semidiameters, and ſines, of like arches one to another.

And therefore the proportion betwixt the Meridian and a parallel, or betwixt a degree of the Meridian, and a degree of that parallel, is as betwixt their Semidiameters.

So that if the Semidiameter of the Meridian be taken for the Radius, then the Semidiameter of any parallel will be equall to the ſine of the Complement of that parallels diſtance from the Equinoctiall,

Equinoctiall, in the like knowne parts as the Radius was to

And therefore

As the Radius, to the sine of the Complement of the latitude, or of that parallels distance from the Equinoctiall: So is the Semidiameter of the Meridian, in knowne parts, to the Semidiameter of that parallel in like knowne parts.

Or by changing of the middle termes

As the Radius to the Semidiameter of the Meridian: So is the sine of the Complement of the latitude, to the Semidiameter of that Parallel.

Now euery parallel in this projection being equal to the Equinoctiall, and consequently the degrees of euery parallel being also equall to the degrees of the Equinoctiall, the Meridian, and the degrees thereof, must of necessitie be inlarged, and increase from the equinoctiall towards either pole; to retaine the same proportion that is betwixt the Meridian, and the parallels of the Globe.

For if the sine of the Complemẽt (or the Semidiametes of any parallel) which is alwayes lesse then the Radius or Semidiameter of the meridian, be made equall to the Radius or that Semidiameter: Then that Radius or Semidiameter of the parallel, shall haue such proportion to the secant of that parallels distance from the Equinoctiall, as the sine of the Complemens should haue had to the Radius, becausse

The Radius is a meane proportionall Number betwixt the sine of the Complement of any Arch, and the secant of that Arch.

And therefore as the sine of the Complement, is to the Radius: So is the Radius to the secant of the Arche giuen. & Contrarily

As for Example.

If I would know the proportion betwixt the Meridian, and the parallel of 50 deg. or betwixt a degree of the Meridian, and a degree of that parallel in minutes or miles: I say according to the proportion of the Globe,

As 10000. the Radius to 6417. the sine of 40. deg. (being the Complement of 50. deg. the parallel giuen) So is 60. minutes or miles, answering to a degree of the Meridian, to 38. minutes or miles, answering to a degree in the parallel of 50. d,

Or, if I take ſaid, according to Mr. *Wrights* proiection,

As 15557. (being the ſecant of the latitude) is to 10000. ſo is 60. deg. to 50. deg. 15. min. which is all one with the former worke.

The reaſon hereof is, that if you haue three Numbers in continuall proportion, that is as the firſt to the ſecond, ſo is that ſecond to the third; you may by hauing any two of them (ſo the ſecond be one) and a third Number giuen, finde a fourth Number, in ſuch proportion to the third as the ſecond was to the firſt. As for example. Let 4. 6. and 9. be three Numbers in continuall proportion, and 12. be another Number giuen. Then you may ſay as 4. to 6. ſo is 12 to 18. Or

As 6. to 9. ſo is 12 to 18. becauſe 6. is a meane proportionall Number betwixt 4. and 9.

In like manner, if I am to ſay, as the ſine of the Complement to the Radius; I may ſay, as the Radius to the ſecant of the arch giuen, and whatſoeuer Number ſhall be giuen for the third, the anſwere will be ſtill one and the ſame.

But of the proportionall that is held in the inlarging of the degrees of the Meridian from the Equinoctiall towards either pole, Mr. *Wright* himſelfe hath demonſtrated the ſame in the Errors of Nauigation by the tables of latitude, which he hath calculated by continuall addition of the ſecants: where you may more at large ſatisfie your ſelfe touching that argument.

Now followeth the queſtions themſelues to be performed Arithmetically as viz.

1 By the latitudes and longitudes of two places giuen, to finde the Rumbe or point of the compaſſe of bearing: and their Rumbe diſtance.

2 By the diſtance and latitudes of two places giuen, with the longitude of one of them: to finde their Rumbe, difference of longitude, and the longitude of the other place.

3 By the Rumbe and latitudes of two places giuen, with the longitude of the one place: to finde their diſtance, difference of longitude, and the longitude of the other place.

4 By the longitudes Rumbe, and one latitude giuen, to finde the other latitude and their diſtance.

5 By

5. By the Rumbe, the diſtance and one latitude and longitude giuen; to finde the other latitude, and their differences of longitude, and conſequently the other longitude.

For the better and more eaſie vnderſtanding the reſolution of theſe or the like queſtions, It is firſt neceſſary to knowe of two places giuen, whether lyeth more Southerly, or Northerly, Eaſterly, or Weſterly.

All latitude on the terreſtriall Globe is accompted from the Equinoctiall towards either pole, being numbred in the Meridian from 1. to 90. deg. and taketh the denomination according to the pole, towards which it is numbred: that is either Northwards or ſouthwards. And therefore if both places lye on the North-ſide of the Equinoctiall, the leſſer latitude lyeth more Southerly, and the greater latitude lyeth more Northerly, the difference of latitude being the remainder of thoſe two numbers when the leſſer latitude is taken out of the greater.

And contrarily, if both places lye on the South ſide of the Equinoctiall, the greater latitude lyeth more Southerly, and the leſſer more northerly; the difference of latitude being found as before.

If one place lye vnder the Equinoctiall, and the other without it; that without the Equinoctiall lyeth more northerly or ſoutherly according to the denomination of the latitude of that place, the difference of latitude being the latitude giuen.

And laſtly, if one place lye on the north, and the other on the ſouth ſide of the Equinoctiall; that on the ſouth ſide lyeth more ſoutherly; And that on the north ſide more northerly: the difference of latitude being the ſumme of both latitudes added together.

Againe, all longitude on the terreſtriall Globe, is accompted from ſome fixed Meridian into the Eaſt, being numbred in the Equinoctiall or ſome circle parallel vnto it, from 1. to 360. degrees. And therefore of two places giuen, differing in longitude, the greater longitude lyeth more eaſterly, and the leſſer longitude lyeth more weſterly; except (accompting from the leſſer to the greater longitude) they are more then 180. degrees diſtant, for then the leſſer longitude lyeth more eaſterly, and the greater longitude lyeth more weſterly: the difference in

longitude being the remainder when the leſſer number is taken out of the greater; but if that remainder exceed 180. deg. then that exceſſe taken from 360. deg. leaueth the difference of longitude.

Hereby is appeareth that the limits or bounds of North and South are the poles themſelues: but of Eaſt and Weſt there are no limits.

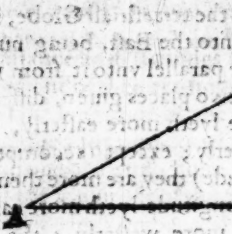
Pro: 1. *To finde how many Miles altereth a degree of the Meridian; or ſerueth to raiſe or depreſſe the pole one degree vpon any Rumbe or point of the Compaſſe giuen.*

The legment of the Meridian, and the legment of any Rumbe included betwixt any two parallels of like diſtance, are in one and the ſame proportion one to another in all latitudes. And therefore by the firſt Axiom of the third of *Pitiſc*: As the Radius is to the ſecant of the angle, included betwixt the Meridian and the Rumbe giuen: So is the miles or minuts anſwering to a degree of the Meridian betwixt any two parallels: to the miles or minutes altering a degree of latitude (or raiſing or depreſſing of the pole one degree) vpon the Rumbe or point of the Compaſſe giuen.

Or elſe I might ſay, As the ſine of the angle differing from the parallel is to the Radius: So is the miles or minutes of one degree of the Meridian: to the miles or minutes that I am to ſaile vpon that point, to alter one degree of latitude.

Example.

I demaund how many Miles I ſhall ſaile to alter one degree of latitude vpon an E.N.E. W.N.W. E.S.E. or W.S.W. Rumb?



In

In the triangle A B C. Let B C. represent 60'. or a degree of the Meridian: A B C. the angle given different from the Meridian, either East or West; whose complement is the angle B A C. from the parallell or East and West line, either to the Northward or Southwards: that is A B C. is an angle of 6. points of the compasse, or 67. deg. 30'. from the meridian, and B A C. is an angle of 2. points, or 22. deg. 30'. from the parallell, accompting for euery point of the compasse, 11. deg. 15'. And lastlie, let the angle A C B. be an angle of 90. deg. or a right angle: because the meridian and the parallell cut one another at right angles, A B. representing the parallell, or east and west line, and let A B. be the line sought for, Then I say.

As C B. the Radius 10000. *to B A.* 26131. *the* Secant *of the angle A B C.* 67 *d.* 30'. *So is C B.* 60 *Miles or one degree of the Meridian: to B A.* 156. $\frac{716}{1000}$ *miles: that I shall saile vpon that point of the compasse, to alter one degree of latitude: Or,*

As B C. 3826. *the sine of the angle B A C.* 22. *deg.* 30'. *to* 10000 *the Radius B A. So is B C.* 60. *miles: to B A.* 156. $\frac{716}{1000}$ *miles: as before.*

And this rule, is generally held, aswell vpon the ordinary Chart, as on the Globe, or a Mappe made after M. *Wrights* projection. Wherein you are to note that if the course be northerly, you shall raise or eleuate the pole: And contrariwise if the course be southerly, you shall lay or depresse the pole, in the north latitude. But if you be to the southward of the Equinoctiall, and your course southerly, you shall raise the pole: and depresse it when your course is northerly.

Pro: 2. To finde the distance betwixt 2 places lying in a parallell, that is East and West, one from another: their longitudes and latitude being given.

Multiplie their difference in longitude by the miles answering to a degree of longitude in that parallell, the product wilbe the distance required: Or else,

Multiplie their difference in longitude by 60. miles, answering to a degree of the equinoctiall: And then either,

As the Radius is to the sine of the complement of the latitude giuen: So is the difference of longitude multiplied by 60. to the distance required. Or,

As the Secant of the latitude to the Radius: So is the difference of longitude multiplied by 60. to the distance in miles, as before.

Example.

Let the southermost parte of the Iland of S. Maries, one of the Azores: and Cape S. Vincent, both lying in the latitude of 37. d. be 2. places whose distance is required. And admit their longitudes to be as followeth

S. Maries Iland longitude	351. *deg.*	10'.
Cape S. Vincent longitude	007.	10'. Which
Substracted, The remainder is	344.	Which
Againe substracted from 360. *deg. resteth*	16.	*For*

their difference of longitude: which multiplied by 47. [illegible] Miles, being the miles answering to 1. degree in the parallell of 37. degrees 35 before taught, produceth 766. [illegible] Miles, for their parallell distance. Or,

As the Radius 10000. *so* 7986. *the sine of the complement of* 37. *deg. the latitude giuen: so is* 960. *the difference of longitude multiplied by* 60. *to* 766. [illegible] *miles, the distance required. Or,*

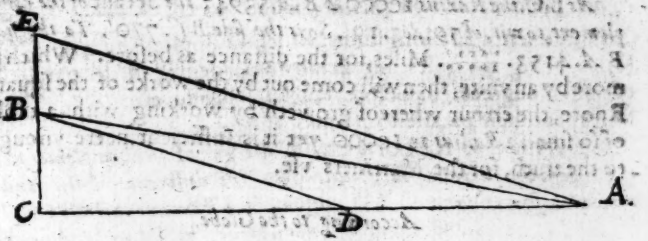
As 12521. *the* Secant *of* 37. *deg. being the latitude giuen to* 10000, *the Radius: So is* 960. *the difference of longitude multiplied by* 60. *To.* 766. $\frac{10504}{13521}$. *miles, the distance as before.* All which workes doe agree whereas in the ordinary Chart, their distance will be found to be 960. miles, which is more then the truth by almost 194. miles, and would haue differed much more, if the 2. places giuen had beene further distant from the Equinoctiall line. But if it be demaunded, whether of the 2. places lye more easterly; by the rules aforesaid, Cape S. Vincent is found to lye more Easterlie then S. Maries Iland, by so much as their difference of longitude is.

Prac: 3. The longitudes and latitudes of 2. places being giuen, both on the one side of the Equinoctiall, to finde their bearing and distance.

Let the Lizard in Cornewall, and an Island lying in the mouth of *Lumleys* Inlet, in *freto Danico* be the two places giuen, And let their course and distance be required: Admit the Latitude and Longitudes of those two places, to be as followeth:

Lumleys Inlet Latitude, No. 63. deg. 00'. Long. 309. deg.
The Lizard Latitude No —— 50. 10. *Long.* —— 17.
Their difference in Latitude —— 12. 50'. differ: lo: 68.

Which is 770'. for the difference of Latitude, and their difference of longitude is 4080'. both their differences of latitude and longitude, being multiplied by 60'. as is vsuall vpon the ordinary Chart, according to which wee will first worke.



According to the ordinary Chart.

In the right angled Triangle, A B C. let B. represent *Lumleys* Inlet; and A. the Lizard point: in which Triangle according to the ordinary Chart, are the 2. lines including the right angle, B C A. giuen, together with the right angle B C A. to wit, B C. 770. minutes the difference of latitude. And C A. 4080. min. the difference of longitude; Then I say by the second Conſ: of the first Axiome of the 3. booke of *Pitiſcus*,

As B C. 770'. *the difference of latitude to* C A. 4080'. the difference of longitude, *So is the Radius* 10000. *to* 52987. *the* Tangent *of* 79. *deg.* 19'. *for the acute angle* A B C. Whose complement is 10. deg. 41'. *For the other* acute angle B A C. Whereby I conclude the bearing of *Lumleys* Inlet from the *Lizard* to be 10. deg. 41'. from the west northwards, that is almost W. by N. And the bearing of the lizard from *Lumleys* Inlet to be 79. deg. 19'. from the south eastwards, that is, E. by S. and 34'. to the eastward.

Now for their distance, you may finde it by the square Roote, by extracting the square roote out of the summe of the two squares of the sides giuen, or else by the 2. Axiome of the 3. of *Pitiscus*: thus,

As B C. 1854. *the sine of the Angle* B A C. 10. *deg.* 41. *to* B A. *the Radius* 10000. *So is the side* B C. 770. *miles to the side* B A. 4153. $\frac{3}{10}$ *miles for the distance required. Or by the second briefe* Rule of the 5. booke.

As B C. *the Radius* 10000 *to* B A. 53943. *the* Secant *of the complement, to wit, of* 79. *deg.* 19'. *So is the side* B C. 770'. *To the side* B A. 4153. $\frac{3}{10}$. Miles, for the distance as before. Which is more by an vnite, then will come out by the worke of the square Roote, the errour whereof groweth by working with a table of so small a *Radius* as 10000. yet it is sufficient neere ynough to the truth, for the Marriners vse.

According to the Globe.

When B C. is giuen 770'. for the difference of latitude vpon the Meridian, and C A. 68. deg. or 4080'. for the difference of longitude in the middle parallell betwixt those two places, the same line C A. must be foreshortened in such proportion as the *Radius* is to the sine of the complement of the middle parallel, by finding the sine of the complement of the middle parallell, in this manner. By the 2. Pro: I say.

As 10000. *the Radius to* 4540. the sine of the complement of 63. deg. to wit, 27. deg. So is 4080. to: 1852. the miles answering to the difference of longitude in the parallell of 63. deg. Againe I say.

As

As 10000. *the Radius to* 6406. the sine of the complement of 50. deg. 10. to wit, 39. deg. 50′. So is 4080. to 2613. the miles answering to the difference of longitude in the parallell, of 50. d. 10′. So is found,

The miles answering to the difference of Longitude in the parallell of 63. deg. } 1852. Miles
And in the parallell of 50. deg. 10′. is —— 2613. Miles

The somme whereof is — 4465.
½. Thereof is —— —— 2232. ½. for the line CD. representing now the difference of longitude in the Triangle BCD. Or to finde the line CD. more briefly at one worke,

The sine of the complement of 63. deg. to wit, of 27. d. is 4540
The sine of the comple. of 50. d. 10′. to wit, of 39. 50′. is 6406
The totall is — 10946
½. Whereof is — 5473

Then I say: As 10000. the *Radius*, to 5473. taken heere for the sine of the complement of the middle parallell; So is 4080. to 2232. for the line C *D*. as before.

Now in the right angled triangle, B C D. I haue the 2. sides giuen, comprehending the right angle, to wit, the side B C. 770′ and the side C D. 2232. Wherefore I say: As B C. 770. to *C D*. 2232. so is B C. the *Radius* 10000. to *C D* 28987. the *Tangent* of 70. deg. 58′. for the acute angle, *C B D*. whose complement is —— 19. d. 2′. for the other acute angle B D C.

Yet I may worke (by Compounding the proportions) more briefly. For whereas I saide before,

As 10000. to 5473. So is 4080. to another Number: and As 770 to the other Number: So is 10000. to ỹ Number sought For: I may by omitting the two *Radij* say.

As 770. to 5473. So is 4080. to 28999. the *Tangent* of 79. deg. 58′. for the acute angle, *C B D*. as before.

So that the bearing of the Lizard from *Lumleys* Inlet, is hereby found to be 79. deg. 58′. from the South eastwards: which diuiding by 11. deg. 15′. is 6. points 3. deg. 28′. that is E. S. E. and 3. deg. 28′. towards the East; And the bearing of *Lumleys* Inlet

Inlet from the Lizard is W. by N. and 7. deg. 47'. towards the North.

Now hauing the 3. angles and the 2. ſides comprehending the right angle in the triangle B C D. to wit, B C. 770. miles, and C D. 2232. miles, I may finde the third ſide, B D. as was taught according to the plaine Chart, in the former part of this Propoſition: viz. either by the extracting the ſquare Roote out of the ſumme of the ſquares of the two ſides; or by the ſecond briefe rule, of the fifth booke of *Peteſie*. For,

As 10000. the Radius to 30664. the Secant of the angle C B D. 70 deg. 58'. So is the ſide B C. 770. miles to the ſide B D. 2361. miles, for the diſtance ſought for. Or els

As B C. 3261. the ſine of 19 deg. 2'. is to B D. the Radius 10000. So is the ſide B C. 770. miles, to the ſide B D. 2361. miles, for the diſtance as before.

Aecording to the true Sea-Chart.

But ſuppoſe C A. the difference of longitude to be 4080. miles of the Equinoctiall, or of any parallel equall vnto it, as all the parallels are equall thereunto in a Chart after Mr. *Wrights* proiection: then cannot the line C B. repreſent the true difference of latitude: but muſt be inlarged according to the proportion that is betwixt the Equinoctiall, and the middle parallel betwixt the latitudes giuen, which although it be not preciſely true according to Art, for that the Sines, Tangents and Seacants, doe not differ by equall proportion, yet it is ſufficient neere enough for the Marriners vſe, and ſuch as haue not Mr. *Wrights* booke, to take this way. And it is thus performed:

Firſt adde the Secants of both the latitudes giuen, and of that Number take the halfe.

Then ſay. As the Radius to that halfe which is here taken for the Secant of the middle parallel. So is the difference of latitude in equall parts giuen, to another number in like equall parts, which ſheweth the line C E.

As for

As for Example.

The Secant of 63 deg. is, —— —— 22027
The Secant of 50. 10'. is, —— —— 15611
which added together, maketh —— 37638
the ½ whereof is —— —— 18819 for the Secant of the middle parallel.

Then I say. *As the Radius* 10000. *to* 18819. *So is* 770. *the difference of latitude when C D. is taken for the difference of longitude. To C E.* 1449. *when C A. is taken for the difference of longitude*: So haue I another Triangle A E C. which is equiangled to the Triangle A B C. and therefore their sides are proportionall by the 46. of the first booke of *Pitiscus*. And by this way to finde their bearing and distance, I say,

As the side A C. 4080'. *to the side C E* 1449. *So is the Radius* 10000. *to* 3551. the Tangent of 19 deg. 33'. for the Acute angle E A C. whose Complement is 70. 27'. for the other acute angle C E A.

And for their distance it is found as is before set downe to be 2301. miles.

Yet their bearing and distance, may be found more exactlie then by any of the former workes, by the helpe of the table of latitudes, calculated by Mr. *Wright* as afore said, after this manner.

Take the difference of the Meridionall parts answering to the latitudes giuen, for the side C E. and for the other side A C. take the difference of longitude in miles, and multiply that by 10. Then say as C E. the difference of latitude in equall parts is to C A. *the difference of longitude in miles, multiplyed by* 10. *So is the Radius to the Tangent of the angle A E C.*

As for Example.

The Meridionall parts in that table for 63. deg. is — 49053
And for —— 50. 10 is — 34901
The difference is —— 14152. for the side E C. Againe, the difference of longitude in miles, is 4080. which multiplyed by 10. maketh 40800. for the side A C

Then I say. As E C. 14152. to A C. 40800. So is E C. 10000 the Radius. To A C. 28829. the Tangent of 70 d. 53'. for

for the acute angle A E C. whoſe complement is 19 deg. 7', for the other acute angle C A E. Then to finde their diſtance, I ſay as before taught.

As C E. the Radius 10000. to E A the Secant of 70 deg. 53'. to wit, 30535. So is C E 770. the difference of latitude in miles to E A. 2351. $\frac{19\,5\,5}{1\,0\,0\,0\,0}$ miles, for their true diſtance vpon that Rumbe.

Now let vs compare theſe workes together, and ſee their difference, taking the laſt worke for the truth; becauſe it is wrought by tables calculated, to euery minute of the Meridian, whereas the former workes are wrought, without the helpe of thoſe tables.

By the ordinary Chart, the bearing of the Lizard from *Lumleys* Inlet, is 79 d. 19'. from the S. Eaſtw: the diſtance 4153'. ỹ true bearing 70. 53 from the S. Eaſtw: true diſtance 2351. ỹ diff: to much 08 26 to the Eaſtw: diſtance, too much 1802. miles.

By working by the ſine of the Complement of that middle parallel, the bearing of the Lizard from *Lumleys* Inlet is ——— 70.d. 58'. from ỹ S.Eaſtw: ỹ diſtance 2361
the true bearing 70 53. from ỹ S.Eaſtw: the diſtance 2351
ỹ differẽce to much -- 05. to the Eaſtw: diſtãce to much 0010

By working by the Secant of the middle parallel, the bearing of the Lizard from *Lumleys* Inlet is as followeth.

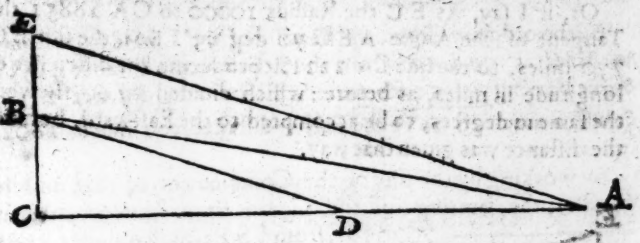
——— 70. d. 27'. from the S.Eaſtw. the diſtãce 2301
true bearing 70, 53. from the S.Eaſtw. the true diſt. 2351
the difference --- 26'. to litle to the Eaſtw. diſt. to litle 050 miles.

So that hereby it appeareth that any of the former workes is ſufficient nere enough for the Maryners vſe; onely the plaine Chart is to be reiected, for that it differeth from the truth in the bearing more then $\frac{1}{4}$ of a point of the compaſſe. And in the diſtance it bringeth out to much by 1802. miles.

Pro. 4. The Latitudes of two places being giuen, with the Longitude of one of them, either Eaſtward or Weſtward, and their diſtance: To finde their bearing and difference of Longitude. And thereby the Longitude of the ſecond place.

Let *Lumleys* Inlet and the Lizard as afore be the two places giuen; And let the latitude and longitude of *Lumleys* Inlet be giuen for the one place, and the latitude of the Lizard for the other place, together with their true diſtance 2351. miles.

Lumleys Inlet latitude 63. deg. and longitude 309. deg. the latitude of the Lizard 50. 10'. Nor: long. their diſtance to the Eaſtward is, 2351. miles.



According to the ordinary Chart.

In the Triangle A E C. let the ſide E C be giuen 770'. for the difference of latitude in the ordinary Chart, and let the ſide E A. 2351. miles in the ſame Triangle be giuen for the diſtance; And let the acute angles A E C. and C A E. together with the line C A. be demaunded. I ſay,

As C E. 770. miles, to E A. 2351. miles. So is C E the Radius 10000. to E A. 30532. the Secant of 70 deg. 53'. for the acute angle A E C. whoſe Complement is 19 deg 7'. for the other acute angle C A E. Then becauſe the diſtance was giuen Eaſtward, I conclude the bearing of the Lizard from *Lumleys* Inlet, to be 70 deg. 53'. from the South Eaſtwards; And I ſay from the South, becauſe the latitude of the Lizard is the leſſer latitude.

Againe,

Againe, for the difference of longitude, it may be found either by the ſquare Roote, thus:

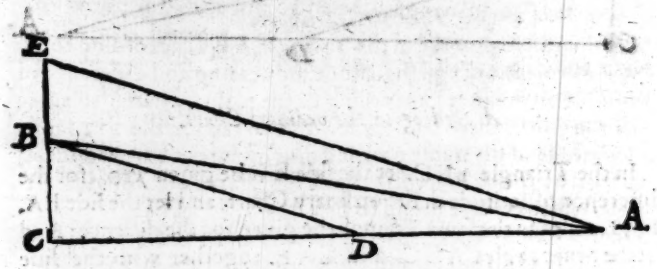
Subſtract the ſquare of 770 out of the ſquare of 2351, the ſquare Root of the remainder is the difference of longitude in miles, which diuided by 60, ſheweth the ſame in degrees.

As for Example.

The ſquare of 2351 E A, is ——— 5527201 from whence
The ſquare of 770 E C, ——— 592900 ſubſtracted,

The remainder is the ſquare of C A, 4934301 whoſe ſquare Root is 2221 miles, for the ſide C A. Which diuided by 60, giueth 37, deg. 01 '. for the difference of longitude; So that if I adde 37 deg. 1'. to 309 deg. the longitude of *Lundays* Inlet; I ſhall haue 346 deg. 1'. for the longitude of the Lizard.

Or, if I ſay, As E C the Radius 10000 to C A 28851 the Tangent of the Angle A E C 70 deg. 53'. So is the ſide E C 770 miles, to the ſide C A 2221. It produceth the difference of longitude in miles, as before: which diuided by 60, ſheweth the ſame in degrees, to be accompted to the Eaſtward, becauſe the diſtance was giuen that way.



According to the Globe.

Againe, In the Triangle B C D. let the ſide B C. be giuen, 770'. for the difference of latitude according to the Globe. And let B D. 2351. miles in the ſame Triangle be giuen for the diſtance: And let the demaund be as before.

I ſay. As B C. 770. miles, to B D. 2351. miles: So is B C. the Radius 10000. To C D. 30532, the Secant of 70 deg. 53'.

as before

as before it was found by the plaine Chart. And likewise, for the difference of longitude, it is found in miles after the same way: by saying,

As B C. the Radius 10000. to C D. 28851. the Tangent of the angle D B C. 70 deg. 53'. So is the sine B C 770. miles to the side C D. 2221. miles, for their difference of longitude in miles: But because these miles or minutes are answerable to so many minutes of the middle parallel betwixt the latitudes given, in such proportion as is betwixt the Semidiameter of the middle parallel and the Equinoctiall: Therefore first, having found, by the 3 *Pro.* the sine of the complement of the middle parallel, to be 5473.

I say, *As* 5473. *to the Radius* 10000. *So is* 2221. *minutes of the middle parallel to* 4058. *minutes of the Equinoctiall.* Which diuided by 60. giueth 67. degrees 38', for their difference of longitude; which 67 deg. 38', added to 309 d. the longitude giuen, the totall is 376 deg. 38', from whence 360. deg. be taken away, the Remainder is 16 deg. 38', for the longitude of the Lizard.

The true Sea-Chart.

Againe, If you worke in the Triangle A E C, according to M. *Wrights* Proiection, you shall finde the bearing and difference of longitude in minutes (according to the plaine Chart) to agree with the worke there set downe. But to reduce the 2221 min. of longitude of the midle parallell into minutes of the Meridian; you shall by the 3. *Proposition* finde the Secant of the middle parallell, which is there found to be 18819; Then,

As the Radius 10000 to that Secant, So is 2221 of the middle parallell to 4179 minutes of the Meridian: Which 4179 deuided by 60'. giueth 69 deg. 39'. for the difference of longitude, whereby the longitude of the Lizard is found to be 18 deg. 39'.

Yet againe, more nere by the help of the Table of Latitudes. First finde the difference of the Meridionall parts, by the 3 *Pro.* which is 14152: Then after you haue found the 2 acute Angles as afore, viz; A E C 70, deg. 53', and C A E 19, deg. 7', you may say,

As 100000 E C, being the Radius multiplyed by 10, is to

C A 28830 the Tangent of the angle A E C 70 deg. 53'. So is C A 14152 the difference of the Meridionall parts, to 4080 the ſide C A, for the minutes or miles of the difference of longitude, which being deuided by 60, giueth 68 deg. for the difference of longitude, and thereby the longitude of the Lizard is found to be 17 deg.

Thus you may perceaue that in this queſtion, ſuppoſing the latitudes and diſtance to be true, there is no difference in finding the Rumbe or bearing, in any of theſe 3 operations, either by the ordinary Chart, the Globe, or Mappe. The only difference is in the Longitude. For

The true difference in longitude by the tables of latitude, is,	68 deg.	
By the ordinary Chart the difference of longitude is	37 deg.	
which is too little by	31 deg.	
By the ſine of the Comple: of the middle parallel diff:	67.	38'.
which is too little by	00.	22'.
By the Secant of ½ middle parallel ½ diff in longitude	69.	39
which is too much by	01.	39

Pro: 5. The latitudes of two places being giuen, with the longitude of one of them and their bearing; to finde their diſtance, difference of longitude, and conſequently the longitude of the Second place.

Let the Lizard and *Lumleys* Inlet be the two places giuen, as before: And let the latitude and longitude of the Lizard, And the latitude of *Lumleys* Inlet, together with their bearing be giuen: And let their diſtance &c. be required.

Lumleys Inlet, latitude Notth 63. deg. longitude ——

The Lizard, latitude North 50. 10. longitude 17 degrees

The bearing of *Lumleys* Inlet from the Lizard 19. deg. 7'. from the Weſt northward, that is 70 deg. 53', from the North weſtwards.

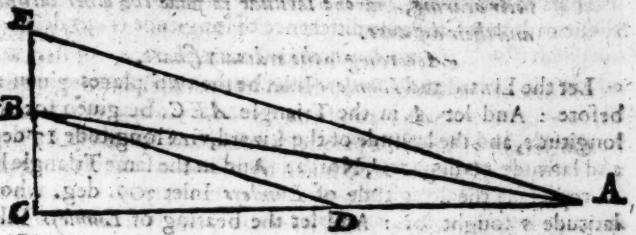
According to the ordinary Chart.

In the Triangle A E C. let the ſide E C. be giuen, 770. miles for the difference of latitude, together with the acute angles

A E C.

A E C. 70 deg. 53'. and *E A C.* 19 deg. 7', And let the sides *A E.* and *A C.* be sought for: First I say,

As E C. 10000. *the Radius to E A* 30535. *the Secant of the angle C E A. So is the side E C.* 770'. *to the side E A.* 2351 *miles, for the distance required.* Which being had, their difference in longitude is found by the 4. Pro. to be 37. deg. 1'. which for that the bearing is westward, is to be taken from the longitude, giuen, viz. 17 deg. Now I cannot take 37 deg. 1', from 17. d. And therefore I take it from 360 deg. and 17', that is 377 deg. the remainder is 339 deg. 59'. for the longitude of *Lumleys* Inlet, being the thing required.



According to the Globe.

In the Triangle *B C D.* Let *B C.* represent, according to the Globe, the difference in latitude 770', and let the acute angle *C B D.* 70 deg. 53', and the acute angle *B D C.* 19 deg. 7', be also giuen for their bearing. And let the distance &c. be demaunded: First I say,

As the Radius B C. 10000. *to* D *B.* 30535. *the Secant of the angle CD B.* 70 *deg.* 53', *So is the side B C.* 770'. *to the side B D.* 2351'. *miles, for the distance required.* And afterwards for the difference in longitude. It is found by the 4. *Pro:* to be 67. deg 38', which substracted from 360 deg and 17 deg. that is from 377 deg. the remainder is 309 deg. 22', for the longitude of *Lumleys* Inlet.

The true Sea-Chart.

Againe, In the triangle *A E C.* as before taught in the ordinary

Chart, the diſtance will be found to be 2351. miles: and the difference of longitude by the 4. Pro: to be by working by the Secane of the middle parallell, 89 deg. 39'. for the difference of longitude, which taken from 377. deg. the Remainder is 287 deg. 21'. for the longitude of *Lumleys* Inlet.

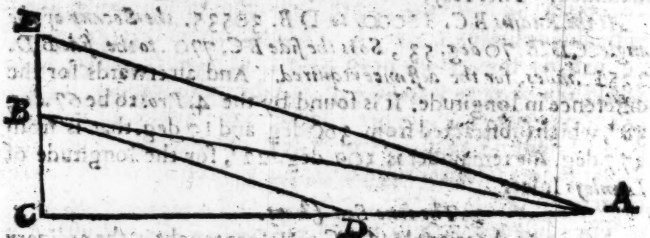
But if you worke by the table of latitudes as is ſet downe in the laſt *Pro:* the difference of longitude will be 68. deg. which taken from 377. deg. the Remainder is 309 deg. for the true latitude of *Lumleys* Inlet.

The difference of theſe workes is only in the differcene of longitude as is ſet downe in the laſt *Pro:* aforegoing.

Pro: 6. The longitudes of two places being giuen; together with their bearing, and one latitude to finde the other latitude and their diſtance.

According to the ordinary Chart.

Let the Lizard and *Lumleys* Inlet be the two places giuen as before: And let *A.* in the Triangle *A E C.* be giuen for the longitude, and the latitude of the Lizard, viz: longitude 17 deg. and latitude 50 deg. 10', North: And in the ſame Triangle let *A.* repreſent the longitude of *Lumleys* Inlet 309. deg. whoſe latitude is ſought for: And let the bearing of *Lumleys* Inlet from the Lizard be giuen 19 degrees 7'. from the Weſt Northwards, that is 70 deg. 53'. from the North Weſtwards: whereby is giuen, the two acute Angles E A C. 19 deg. 17'. & A E C. 70 deg. 53'. Then haue I giuen in the Triangle A E C. ſufficient tearmes for the reſolution thereof, viz: the difference of longitude 68 deg. or 4080'. repreſented by the line A C. and the 3. angles, whereby I reſolue that Triangle as followeth. Firſt according to the ordinary Chart, I ſay:



As

As A C. the Radius 10000 to C E. 3466 the Tangent of 19 deg. 7 : So is A C. 4080 miles to C E. 1414 miles for the difference of latitude, which diuided by 60 giueth 23 deg. 34'. Now becauſe the courſe was giuen to the Northwards, I adde 13 deg. 34'. the difference of latitude found to 30 deg. 10'. the latitude giuen, the ſumme is 73 deg. 44'. for the latitude of *Lumleys* Inlet. Againe for the diſtance I ſay.

As A C. the Radius 10000 to A E. 10583 the Secant of the angle C A E : So is 4080'. the difference of longitude. to A E. 4317 the diſtance required.

And becauſe it is difficult in this propoſition to finde the difference of latitude and diſtance by the bearing, and difference of longitude, becauſe onely one latitude is giuen, whereby I can neither take the ſine of the Complement or Secant of the middle parallell as was done in the former propoſitions, I will therefore onely in this, ſhew the true working of it by the helpe of the Tables of latitude, thus.

First finde the number in the Tables of latitude, anſwering to the latitude giuen: And then ſay, As the Radius to the Tangent of the angle different from the parallell: So is the difference of longitude multiplied by 10 to an other number, which added to the number anſwering to the latitude of the place giuen (if the places latitude ſought for be more Northerly) giueth you the number anſwering in thoſe Tables to the latitude of the ſecond place: Or that nũber found, ſubſtracted from the number anſwering to the latitude of the place giuen, if the latitude demaunded be Southerly, leaueth the number anſwering in the ſame Tables to the latitude of the ſecond place. And therefore,

As A C the Radius 10000 to C E 3466 the Tangent of 19. deg. 7'. So is A C 40800 the difference of longitude in minutes multiplied by 10 to C E 14141 which for that the bearing is giuen Northward, I adde to 34901 the number anſwering in the Tables of latitude to 50 deg. 10'. the latitude giuen, the ſum is 49042. and that number I ſeeke in the Tables of latitude, where I finde it to anſwere to 67 deg. of latitude: Then I ſay the latitude of *Lumleys* Inlet is 63 deg. North, which was required.

The diſtance is alſo found, hauing both latitudes and

longitudes by the 3. or 5. Prop: to be 2351 miles.

The difference of the ordinary Chart in this Prop: from the truth, is in the latitude of the second place, and the distance.

For by the ordinary Chart.

The latitude of *Lundys* Inlet is 73 44'. the distance 4317'
The true latitude is ——— 63 —— true distance 2351

The difference is too much La: 10 44. in distance 1966 miles which is produced in the ordinary Chart more then the truth.

Pro: 7. The latitude and longitude of one place being given, together with the Rumbe and distance, to finde the latitude and longitude of the second place.

Let E. in the Triangle A E C. represent *Lundys* Inlet, whose latitude is given 63 deg. North and longitude 309 deg.

And let A. represent the Lizard whose latitude and longitude is required: let the angle A E C. be given 70 53'. for the bearing from the South Eastwards, And let the distance E A. be given 2351 miles.

According to the ordinary Chart.

First I saye to finde E C. the difference of latitude, and thereby the latitude of the second place.

As A E. 30535. the Secant of the angle A E C. 70 deg. 53', to E C the Radius 10000. So is A E. 2351'. the difference given: to E C. 770', the difference of latitude: or els by bringing the Radius into the first place, by the fourth briefe Rule of the fift of *Pitiscus.*

As A E. 10000. the Radius: to E C. 3274. the sine of the Complement, to wit, 19 deg. 7'. So is A E. 2351'. the distance given to E C. 770'. the difference of latitude. Which diuided by 60', the quotient is 12. deg. 50'. which for that the bearing is given southward, I substract from 63 deg. the latitude given, the Remainder is 50. deg. 10'. for the latitude of the Lizard.

Then

Then by the 5 *Pro.* having the latitudes, and bearing giuen, together with one longitude, you shall finde the difference of longitude by the ordinarie Chart to be 37 deg. 1/. which because the bearing is Eastward, is to be added to 309. the longitude giuen, the totall is 346. 1/. for the longitude of the Lizard.

According to the Globe, or true Mappe.

Now if you worke according to the Globe or true Mappe, for the finding of the difference of latitude, & consequently the latitude of the second place, it is all one with the worke after the plaine Chart.

But for the difference of Longitude, it is found by the seuerall wayes set downe in the fourth *Pro.* where the difference from the truth is set downe: And the true difference of longitude is thereby found to be 68 deg. which added to 309. maketh 377. d from whence 360 deg. being taken, leaueth 17 deg. for the true longitude of the Lizard according to the first assumption.

So that by the resolution of these questions, it may be gathered, that no two places not lying vnder the Equinoctiall or Meridian line, can be truly situated in the ordinary Chart. For if you will situat them by latitude and longitude, their distance will be more then it should be, and the bearing more to the East or West then it ought to be, as appeareth in the 3 *Pro:*

Againe, if you will situate them by their true course and distance, keeping the latitudes true as you ought, the difference of longitude will be lesse then it should be, as appeareth by the 4. 5. and 7. Pro.

And lastly, if you will situate them by course and distance, respecting their longitudes, then the difference of latitude will be more then the truth, as by the 6. Pro: you may perceiue.

All which errors are more grosse and apparant, the further that the two places are distant from the Equinoctiall towards either Pole.

And

And thus much ſhall ſuffice for the reſolution of the former queſtions of the Mappe Arithmetically, which who ſo well vnderſtandeth, may thereby be able to performe any other Nauticall queſtion, that may be reſolued vpon the Mappe without the ſame, by Arithmeticall calculation only, with the help of the Table of latitudes, and the Canon of Triangles.

All which Propoſitions or any other queſtions of right lines, or right angled ſphæricall Triangles, may be performed by the *Circular Scale* without Arithmaticke: The vſe of which Inſtrument is facile, and fitting for all practicioners in the Mathematickes.

TVVO MOST PROFI-TABLE PROPOSITIONS FOR The finding of the variation of the Compasse.

FOR that the finding of the variation of the compasse is of most necessary vse for the Marryners direction in sayling; I haue hereunto added two principall Pro: for the finding of the true amplitude or Azimuth of the Sunne, whereby the variation may b ereadily found out.

THe amplitude of the Sun, called also the bredth of the Suns rising or setting, is the degrees & minutes that the Sun riseth or setteth from the true East or West point in the *Horizon*, and is alwaies of the same denomination that the Suns declination is of.

THe *Azimuth* is the true point of the compasse that the Sun is on, at any height or Almicanter giuen; wherof there are seuerall cases as hereafter shall be set downe: But first for the finding of the true Amplitude by the Latitude and declination giuen.

Data { *Latitude* } { *North* } { 50 d. } I demaund the true Amplitude of ☉ sunes rising
{ *Declination* } { *North* } { 20 d. }

As the sine of the Complement of the latitude: to the sine of the declination: So is the Radius, to the sine of the Amplitude. Or to auoid diuision by the second breife Rule of the 5. of *Pitiscus*.

As the Radius to the Secant of the latitude: So is the sine of the Suns declination, to the sine of the Amplitude. And therefore in this *Pro*:

1 *As* 64278 *the sine of* 40 *degrees, being the Complement of* 50 *deg. the latitude giuen : to* 34202 *the sine of* 20 *deg. the Sunnes declination giuen : So* 100000 *the Radius, to* 53209 *the sine of* 32 *deg.* 9′. *for the Amplitude required, from the East northwards: which diuiding by* 11 *deg.* 15′. *the degrees answering to one point of the compasse, the quotient is two pointes* 9 *deg.* 30′. *that is* E N E *and* 9 *deg.* 30′. *to the northwards, for the true point of the compasse of the Sunnes rising or setting at that time according to the data.* Or,

2 *As* 100000 *the Radius, to* 155572 *the Secant of* 50 *deg. being the latitude giuen : So is* 34202 *the sine of the Sunnes declination, to* 53209 *the sine of* 32 *deg.* 9′. for the Amplitude demaunded as before.

But if the true Amplitude were sought at the Sun setting, then the 32 deg. 9′. found, must be accompted from the West northwards in this Pro:

And if the declination, in this case had beene giuen Southwards, then the Amplitude at the Sunnes rising would haue bene found 32 deg. 9′. from the East, southwards : And the amplitude at the Sunnes setting 32 degrees 9′. from the West southwards, And so for any other. Thus the true Amplitude being found, the variation of the compasse is the difference betwixt that, and the Needles amplitude, which euery Sea-man knowes how to obserue.

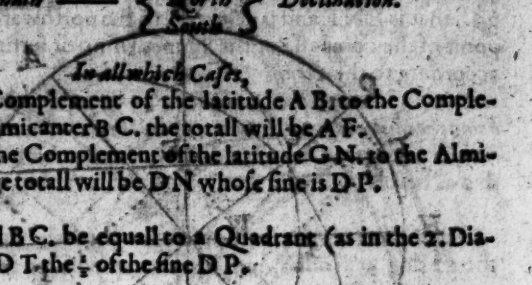
Note, that the greater the Latitude, the greater is the Amplitude; For where the Latitude is equall to the Complement of the declination giuen, both being of one denomination, the Amplitude is 90 deg. from the East or West, because there the Sunne toucheth, as it were the *Horizon*, at the lowest, in the intersection of the *Horizon* and *Meridian* circles.

But where the Latitude is more then the Complement of the declination giuen; both being of one denomination, that is both North, or both South: there the Sunne commeth not at all to the *Horizon*, and so in that respect cannot be said either to rise or sett, for that it is there continually day, so long as the declination is equall to, or more then the complement of the Latitude.

The Latitude declination and Almicanter of the Sun being given: to finde the Azimuth.

This Proposition hath three Cases: For,

Either the Sun hath —— { *No, North, South* } *Declination.*



In all which Cases,

Adde the Complement of the latitude A B to the Complement of the Almicanter B C. the totall will be A F.

Also adde the Complement of the latitude G N. to the Almicanter D G. the totall will be D N whose sine is D P.

1 If A B and B C. be equall to a Quadrant (as in the 2. Diagram) then is D T. the ½ of the sine D P.

2 If A B and B C. be lesse then a Quadrant (as in the third Diagram) the Complement of that summe is F Q. whose sine is F u. which taken from D P. the remainder is D R. the ½ whereof is D T.

3 If A B and B C. be more then a Quadrant (as in the 1. 4. and 5. Diagram) the excesse thereof is F Q. whose sine is F u, which added to D P, the whole is D R. the ½ whereof is D T.

1 Example where the Sun is in the Equinoctiall A B and B C. being allwaies in this case more then a Quadrant.

Data { *Latitude 51 d. 30'. North* / *Almicanter 20.* } *Demaund the Azimuth.*

Let H B G N. be the Solsticiall Colure.

H G. the Horizon.

B E M. the East Azimuth.

A S. the Axis of the world.

Q 2. *Q N. the*

Q N. the Equinoctiall.
F D. the Almicanter.
I K. (in the other Diagrams) the Sunne parallell.
B C. equall to B F or D. and consequently G I. equall to F H or D G. the Almicanter given.



The Worke as followeth.

A B.	38 deg.	30'.	Idem or G N.	38 deg.	30'.		
B C.	70	—	Comple: D G.	20	—		
A F	108	30	D N.	58	30.	D P.	85264
A Q	90						
F Q.	18	30	Whose sine is.			F u.	31730
			The totall is			D R.	116994
			½ whereof is R T or D T.				58497
			From whence substract R P or F u.				31730
			The Remainder is P T or W E.				26767

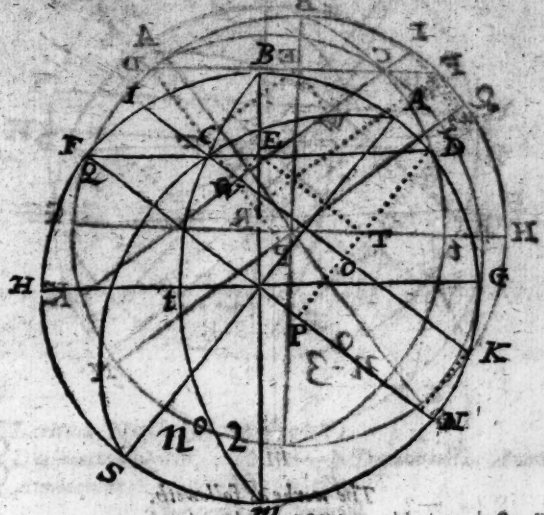
Then.

As D T. 58497 to D E. 100000, So is W E. 26767 to E C. 45758 the sine of 27 deg. 14'. for the true Azimuth of the Sunne from the East Southwards.

2 Example

2 *Example where the Sunne hath North declination A B and B C, being equall to a Quadrant.*

Data { *Latitude North* 51 d. 30 ′ / *Declination N.* 20 — / *Almicanter* 38 30 } *Demand the Azimuth.*



The Worke as followeth.

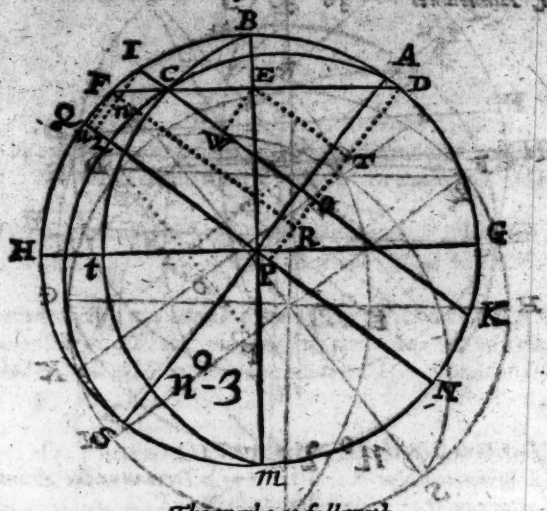
A B 38 deg. 30 ′.	*Idem or G N*	38. 30 ′.	
B C 51. 30.	*Complement D G*	38. 30.	
A C 90.	— *D N,*	77. —	*the*
Sine whereof is		*D P.*	97437
Thereof is P T. or T D,			48718
Substract K r. or O P. the Declynation			34202
Resteth O T. or W E.			14516

Then

As T D. 48718, *to D E.* 100000. *So is W E* 14516, *to E C.* 29796, *the Sine of* 17 *deg.* 20 ′. *for the true Azimuth of the Sun from the East Southwards.*

3 *Example where the Sunne hath North declynation AB. and EC. being lesse then a Quadrant.*

Data { *Latitude* 51 d. 30'. *North:* / *Declynation* 20. — *North:* / *Almicanter* 48. 30. } *Demaund the Azimuth*



The worke as followeth.

AB. 38 *d.* 30'. *Idem or GN.* 38 *d.* 30',
BC, 41. 30. *Compl: DG.* 48. 30.

AF. 80. — *DN:* — 87 — *whose sine DP.* 99862
FQ, 10. *whose sine is Fw, or PR. substracted.* 17364

Resteth DR. — — 82498
½ *whereof is TD. or RT.* — — 41249

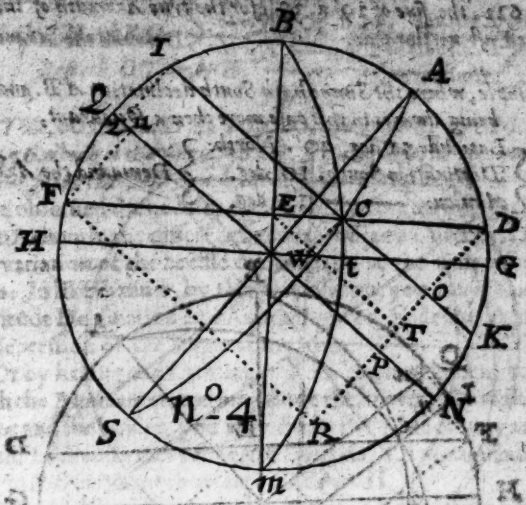
Sine of the declination IZ. 34202. } *Resteth Iw. or RO.* 16838
Substract Fu. — — 17364 }
Which substracted from RT. Resteth OT. or wE. — 24411

Then

As TD. 41249, *to DE.* 100000. *So is WE.* 24411. *to EC.* 59179 *the sine of* 36 *deg.* 17'. *for the true Azimuth of the Sun from the East Southward.*

4 Example

4 Example, where the Sun hath North declination A B. and B C, being more then a Quadrant.



Data { *Latitude North.* 51 d. 30'. / *Declination North:* 20. — / *Almicanter* 10. } *Demand the Azimuth*

The worke as followeth.

A B. 38. 30', *Idem or G N,* 38. 30'.
B C. 80, — *Comple: D G.* 10. —
A F, 118. 30. *D N.* 48. 30. *whose fine is D P.* 74895
A 2. 90. —
F 2. 28. 30. *whose fine is F u. or P R. which is* —— 47715
The whole being added, is D R. 122610
½ whereof is T R. or T D. —— 61305

Sine of the Declina: I z. 34202 }
F u. being added —— 47715 } *Reſteth T O.* } 20612
The totall is O R. —— 81917 } *Or C W, which is* }
Subſtract T R. —— 61305 }

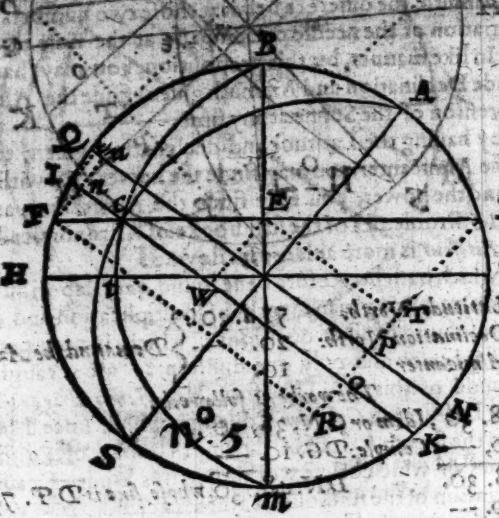
Then

Then

As TD, 61305. is to DE, 100000. So is WE, 20612. to CE. 33622. the sine of 19 d. 30'. for the true Azimuth of the Sun from the East northwards.

5 Example, where the Sunne hath South declination AB. and BC. being Always in this case more then a Quadrant,

Data { Latitude 51 deg. 30'. North: / Declination South. 10 deg. / Amica: —— 15 deg. } Demand the Azimuth



—— The worke as Followeth

AB.	38 deg. 30'.	Idem & GN.	38 deg. 30'.		
BC.	75.	—— Complement DG.	15.		
AF.	113.	30.	DN.	53. 30. whose sine is DP.	80385
AQ.	90.				
FQ.	23.	30. whose sine is FI. or TR. which is			39874
		The totall: DR.			120259
		½ thereof is RT. or TD.			60129

The

The sine E N. is —— 39874 *R T.* 60129
The sine of the decly. I Z sub: 173645
Resteth E N. or R O. —— 22510 *substract* —— 22510
Resteth O T. or W E. —— —— —— 37619

Then

As T D. 60129. *to D E.* 100000. *So is W E.* 37619. *to E C.* 62556. *which is the sine of* 38 *deg.* 43 *'. for the true Azimuth of the Sunne from the East Southwards*; So that hauing at the same time obserued the needles Azimuth, by comparing that with the true Azimuth, the differēce betwixt those two numbers sheweth the variation of the needle or Compasse at the time of obseruation: In like manner, by this proposition, you may (hauing the Latitude Declination and Azimuth giuen) finde the Almicanter or depression of the Sunne at any time.

Or by hauing the Latitude and declynation giuen, together, with the Almicanter, you may finde the hower. Or with the said *Data* and the hower, you may finde the Almicanter; as by the 8. and 16. Problemes of the first booke of Astronomicall questions in *Pitiscus* is more at large set downe.

All which hath heretofore bene found very laborious in the operation, requiring many works of Multiplication and diuision for the resoluing of such questions; which is now performed by *Prostaphericis* and only one diuision as afore taught. The first ground of this Pro: I had from Mr. *Henry Briggs* Mathematicall Lecturer in *Gresham* Colledge, which since I haue applyed to the fourth Axiom of the fourth booke of *Pitiscus*: the excellency of which briefe working in this kinde, I leaue to the consideration of the studious Mathematician.

Much more might haue bene added by way of application to this generall Trigonometry, out of *Pitiscus* himselfe, *Regiomontanus*, *Copernicus*, *Clauius*, *Finkius*, and others; but because my time will not now permit me, I will deferre the same till further occasion be offered, not doubting but in the meane space, some of our English-Mathematicians will hereby take incouragement to publish some workes of their owne, for the benefit of our Country: which I heartily desire, and would be right glad to see effected.

Est voluisse Satis.

Page.	Line.		Error.	Correction.
2	3	for	your	the
5		In the Diagram,	In the Diagram A D E,	D E F.
7	16		right angles	right angled figures
16	2		Read after together	a full point.
17	16		he	the
20	15		quidrilater all	quadrilaterall.
21		last	that the	The
27			GH.	A H.
32	22		concurse	concourse
33			A C.	E C.
34	5		fine: the	fine of the
35	19		parts:	squares
36	31		Multiplied	Multiplicacion
47	11		the sublense	the subtense
51	28		of greater	greater of
60	last but one:		more 1.q.	more 1 ft.
72	33		your	the
61	19		A D.	A C.
82	21.		of the	of halfe the
83	23		and the	and halfe the
	4		differences, second, third &c. of	first second & third differences of
84	20		538	358
	24		44544756	44554746
93	10		faid	fine
94	thruoghout		leaft	left
98	12.		fheweth	fhewed
	17		if	that
102	8		left	leaft
	7		A being the Center	no full point before A, but read the sentence from At
108	5		being	be
	8		excted	excepted
113	5		KD	D
114	8		BGO	GBO
118	15		CB	CD
124	1		fine of the Complement.	Complement
125	13		that	there
126	1.58.9.		R	r
127	21.28.29.30.		B	b
	31 32.		A	a
128	24.25		R	r
129	19 17 18. 19.		B	b
	20, 21.		A	a

Errata.

Page.	Line.	Error.	Correction.
13	13	DR	Dr.
	14	RP	r p
	19	AG	AC
131	7 8.	A	a
	18.19	H	h
	20.22	G	g
132	16.24.26.27	GH	g h
	28	H	h
137	6	by of	of
194	13	IH	FH

In the Nauticall Queſtions.

7	13	AB	AC
16	laſt	CD	BD
17	6	ſine	ſide
18	3	CA	EA
19	10	17, mi.	17 degrees,
	36	67	63
22	24	difference	diſtance,

LONDON.
Printed by *Edw: Allde* for *Iohn Tap*, and are to be ſold at his ſhop at St. *Magnus* corner.
1614.

56500 56500 56500 56500 56500 56500 56500
56539 56539 56539 56539 56539 56539 56539
56567 56567 56567 56567 56567 56567 56567